AGI-Exam

April 2021

Time: 2 h

1 Game Theory

Rock-Paper-Scissors Game (actions R, P, S).
Note for all exercises: Zero sum game; tie if both players play same action.

a) Add extra action X. X wins against R and S, looses against P. Model NF-game with all 4 actions. Find a Nash equilibrium.

b) Redefine utility for action profiles concerned with X such that there exists a pure NE.

c) Redefine utility for action profiles concerned with X such that there exists a strongly dominated strategy and no pure NE
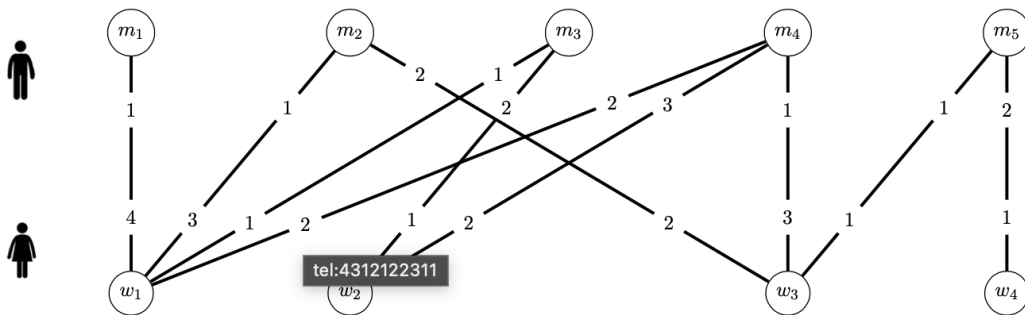
2 Matchings

Figure 1

a) Find a stable matching in a given graph with Gale Sharpley. Write down edges ooft this stable matching and the rejected edges.

b) Show that every stable matching is popular.

c) In the given graph, find a popular matching which is not stable

3   Exercise Auctions

Given single parameter environment: 3 bidders with $v_i \geq 0$ and $X = \{(1,1,0),(1,0,1),(1,0,0),(0,0,0)\}$

a) Find social welfare maximizing DSIC auction (x,p).

b) Let $v_1 \sim U(0,200)$, $v_2 \sim U(0,100)$, $v_3 \sim U(0,50)$. Assume $b_1 = 90$, $b_2 = 70$ and $b_3 = 30$. Compute x and p for the given bids b in a revenue maximizing DSIC auction (x,p).

4   Exercise Congestion Games

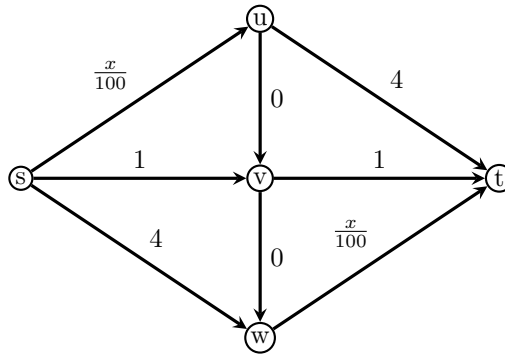Given Atomic routing game with 300 players going from s to t.

Figure 2: Caption

a) Find all Nash flows (reason why they are Nash flows).

b) Find the optimal flow (Hint: total cost is 550).

c) Compute PoA and PoS.

5   Information Elicitation

Consider linear, logarithmic and quadratic scoring rule.

a) Reason about the minumum and maximum output values of all 3 score functions for arbitrary outcomes and distributions. Give examples.

b) Assume that q is uniform distributed over m outcomes. Relate the output of the three scoring rules. Which is largest / smallest?

6   Election

a) Give an example where Minimax-AV fails JR.

b) Give an example where SAV fails JR.

7   True or False

a)

b) Revenue maximizing auction: If all bidders have a valuation from the same strictly distribution, then the highest bidder always gets the item.

c) Does there exist a 2 player NF-Game with only one NE but where both players have a weakly dominant strategy?

d) Does strong diagonalization imply strictly proper?

e) The condorcet looser is always deleted in the first iteration of instant runoff