Prof. Dr.-Ing. Jörg Raisch
M.Sc. Germano Schafaschek
Fachgebiet Regelungssysteme
Fakultät IV Elektrotechnik und Informatik
Technische Universität Berlin
Integrierte Veranstaltung "Ereignisdiskrete Systeme"

Control Systems Fachgebiet Regelungssysteme

Final written exam in course "Discrete Event Systems"

11.03.2021

Maximum score: 40 points

Question 1 (10 points)

Consider a max-plus matrix A, whose precedence graph $\mathcal{G}(A)$ is shown in Figure 1.

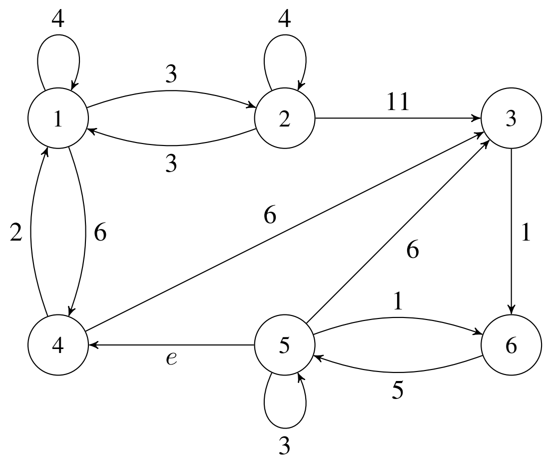

Figure 1: Precedence graph $\mathcal{G}(A)$ for Question 1.

The following powers of matrix A are given:

$$A = \begin{bmatrix} 4 & 3 & \varepsilon & 2 & \varepsilon & \varepsilon \\ 3 & 4 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 11 & \varepsilon & 6 & 6 & \varepsilon \\ 6 & \varepsilon & \varepsilon & \varepsilon & e & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & 3 & 5 \\ \varepsilon & \varepsilon & 1 & \varepsilon & 1 & \varepsilon \end{bmatrix}, \quad A^2 = \begin{bmatrix} 8 & 7 & \varepsilon & 6 & 2 & \varepsilon \\ 7 & 8 & \varepsilon & 5 & \varepsilon & \varepsilon \\ 14 & 15 & \varepsilon & \varepsilon & 9 & 11 \\ 10 & 9 & \varepsilon & 8 & 3 & 5 \\ \varepsilon & \varepsilon & 6 & \varepsilon & 6 & 8 \\ \varepsilon & 12 & \varepsilon & 7 & 7 & 6 \end{bmatrix}, \quad A^3 = \begin{bmatrix} 12 & 11 & \varepsilon & 10 & 6 & 7 \\ 11 & 12 & \varepsilon & 9 & 5 & \varepsilon \\ 18 & 19 & 12 & 16 & 12 & 14 \\ 14 & 13 & 6 & 12 & 8 & 8 \\ \varepsilon & 17 & 9 & 12 & 12 & 11 \\ 15 & 16 & 7 & \varepsilon & 10 & 12 \end{bmatrix},$$

$$A^4 = \begin{bmatrix} 16 & 15 & 8 & 14 & 10 & 11 \\ 15 & 16 & \varepsilon & 13 & 9 & 10 \\ 22 & 23 & 15 & 20 & 18 & 17 \\ 18 & 17 & 9 & 16 & 12 & 13 \\ 20 & 21 & 12 & 15 & 15 & 17 \\ 19 & 20 & 13 & 17 & 13 & 15 \end{bmatrix}, \quad A^5 = \begin{bmatrix} 20 & 19 & 12 & 18 & 14 & 15 \\ 19 & 20 & 11 & 17 & 13 & 14 \\ 26 & 27 & 18 & 24 & 21 & 23 \\ 22 & 21 & 14 & 20 & 16 & 17 \\ 24 & 25 & 18 & 22 & 18 & 20 \\ 23 & 24 & 16 & 21 & 19 & 18 \end{bmatrix}, \quad A^6 = \begin{bmatrix} 24 & 23 & 16 & 22 & 18 & 19 \\ 23 & 24 & 15 & 21 & 17 & 18 \\ 30 & 31 & 24 & 28 & 24 & 26 \\ 26 & 25 & 18 & 24 & 20 & 21 \\ 28 & 29 & 21 & 26 & 24 & 23 \\ 27 & 28 & 19 & 25 & 22 & 24 \end{bmatrix}.$$

a) (2 points)
Determine an eigenvalue λ of A. Is this eigenvalue unique? Justify your answer.

b) (2 points)
Provide the critical graph of A and indicate its maximal strongly connected (m.s.c.) subgraphs.

c) (3 points)
Let

$$Q^+ = Q \otimes Q^* = \begin{bmatrix} e & -1 & -8 & -2 & -6 & -5 \\ -1 & e & -9 & -3 & -7 & -6 \\ 6 & 7 & e & 4 & 2 & 3 \\ 2 & 1 & -6 & e & -4 & -3 \\ 4 & 5 & -2 & 2 & e & 1 \\ 3 & 4 & -3 & 1 & -1 & e \end{bmatrix}, \quad \text{where } Q = \text{inv}_{\otimes}(\lambda) \otimes A\,.$$

Consider the vector $v = [5 \quad 6 \quad 13 \quad 7 \quad 11 \quad 10]'$. Determine two linearly independent eigenvectors of A, say, ξ_1 and ξ_2, for which $\nexists\, \alpha_1, \alpha_2 \in \mathcal{R}$ such that $v = \alpha_1 \xi_1 \oplus \alpha_2 \xi_2$, where $\mathcal{R} = \mathbb{R} \cup \{-\infty\}$.

d) (3 points)
Let a system be represented by a timed event graph whose transitions' earliest possible firing times can be described by $x(k+1) = Ax(k)$, where $x(k)$ is the vector of the k-th firing instants and A is the matrix provided above. Consider the following statement:

"*Regardless of the value of $x(1)$, the system will eventually exhibit a* __________ *behavior.*"

Among the following options, choose the one (and only one) that correctly completes the sentence, and briefly **justify** your answer:

- *nonperiodic*;
- *transient*;
- *1-periodic*;
- *2-periodic*;
- *3-periodic*.

Question 2 (8 points)

Consider the following languages defined over the alphabet $\Sigma = \{\lambda, \beta\}$:

$$L_1 = \{s \in \Sigma^* \mid \nexists\, t, u \in \Sigma^* \text{ such that } s = \lambda t \beta u \beta\}\,;$$
$$L_2 = \{s \in \Sigma^* \mid s \text{ contains the sequence } \beta\lambda \text{ at least twice and does not contain the sequence } \beta\lambda\beta\}\,.$$

a) (4 points)
Provide a nonblocking deterministic finite automaton A_1 such that $L_m(A_1) = L_1$.

b) (4 points)
Provide a nonblocking deterministic finite automaton A_2 such that $L_m(A_2) = L_2$.

Question 3 (10 points)

Consider a robot modeled by the automaton R shown in Figure 2, defined over the alphabet $\Sigma = \{a, a', b, b', c, c'\}$. From its initial position (location I) it can visit three locations, A, B, and C; additionally, it can also visit location B directly from locations A or C. Events a, b, and c represent the arrival of the robot at locations A, B, and C, whereas a', b', and c' represent the arrival back at I when coming from A, B, and C, respectively.

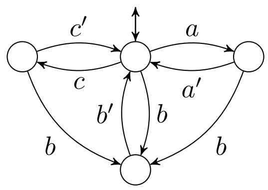

Figure 2: Automaton R modeling the robot for Question 3.

a) (3 points)
Explain, in a short sentence, the meaning of the specification for R represented by the automaton $\widetilde{A}_{\text{spec}}$ in Figure 3, defined over the alphabet $\Sigma_{\text{spec}} = \{a, b, c\}$.

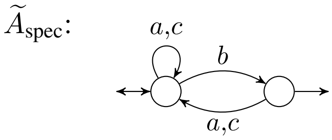

Figure 3: Automaton $\widetilde{A}_{\text{spec}}$ representing a specification for the robot of Question 3.

b) (4 points)
Provide a deterministic finite automaton that represents the following specification:

For every time the robot goes from location A directly to location B, it can only visit location A again after visiting location C at least once.

Indicate also the alphabet over which your automaton is defined.

c) (3 points)
Assume that the sets of controllable and uncontrollable events of R are given by $\Sigma_c = \{a, b, c\}$ and $\Sigma_{uc} = \{a', b', c'\}$, respectively. Is the language $K = \{\varepsilon, abb', cc', bb'cbb'\}$ controllable with respect to the language $L(R)$ generated by R? Justify your answer.
(Hint: you do not need to explicitly construct the language $L(R)$.)

Question 4 (4 points)

Obtain the parallel composition of automata A_1 and A_2 shown in Figure 4, defined over alphabets $\Sigma_1 = \{\alpha, \beta, \delta\}$ and $\Sigma_2 = \{\alpha, \beta, \gamma\}$, respectively.

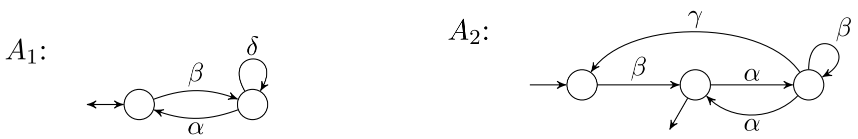

Figure 4: Automata A_1 and A_2 for Question 4.

Question 5 (8 points)

Let the automaton A_{p} in Figure 5 (left-hand side) model the behavior of a plant to be controlled, defined over the alphabet of events $\Sigma = \{a,b,c,d\}$. Assume that events a and c are controllable, whereas b and d are uncontrollable. A specification for the system is represented by a certain automaton $\widetilde{A}_{\text{spec}}$ (not explicitly provided); the automaton $A_K = A_{\text{p}} \parallel \widetilde{A}_{\text{spec}}$ in Figure 5 (right-hand side) is the result of the parallel composition of A_{p} with $\widetilde{A}_{\text{spec}}$. Provide an automaton realization for the least restrictive (and nonblocking) supervisor that enforces the given specification on the plant A_{p}.

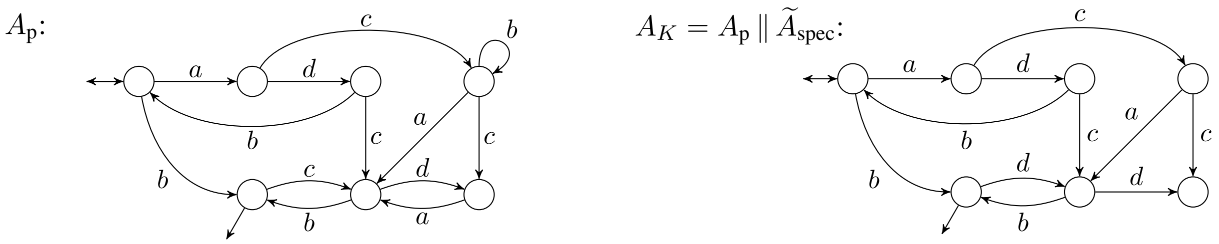

Figure 5: Automata models for Question 5.