$Z = R + jX$, wobei R (Realteil) der Wirk-
widerstand (Resistanz) ist. Frequenzabhängig stimmen Resistanz (Impedanz eines ohmschen Bauelements hat nur einen Realteil).
Der Imaginärteil X steht für den Blindwiderstand (Reaktanz) ... setzt keine Wirkleistung um, ist aber frequenzabhängig; ist jm
Blindwiderstände speichern Energie (Impedanz von C hat nur Im)

1. Thema: Zeiger

$Z = R + jX$ – Impedanzen:
- Kondensator: $Z_C = \frac{1}{j\omega C}$; $U = \frac{1}{j\omega C} \cdot I$
- Spule: $Z_L = j\omega L$; $U = j\omega L \cdot I$
- Widerstand: $U = R \cdot I$ (rein Reell)

für DGL → $j\omega \Rightarrow \frac{d}{dt}$ Bsp: $U_L = L \cdot \frac{d}{dt} i_L$, bzw.: $i_C = C \frac{d}{dt} U_C$

$Z_A = R + jX_A = \frac{\hat{u}(\cos(\varphi_u) + j\sin(\varphi_u))}{\hat{i}(\cos(\varphi_i) + j\sin(\varphi_i))} = \frac{\hat{u} e^{j\varphi_u}}{\hat{i} e^{j\varphi_i}}$

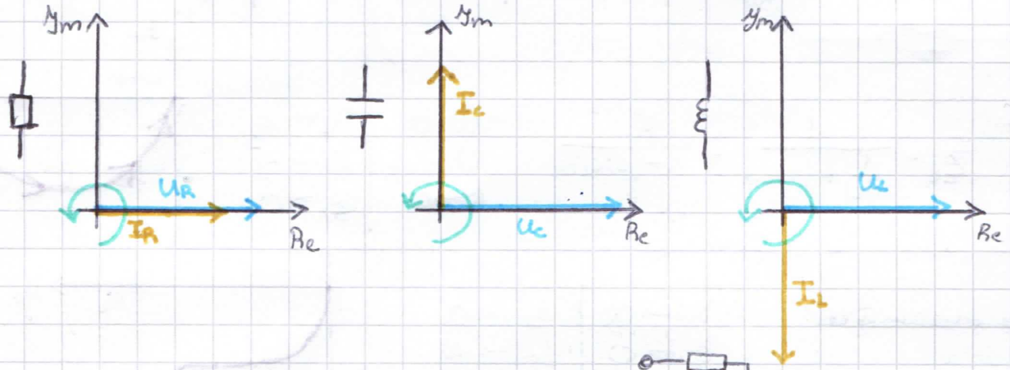
$U_0 = \hat{U}$; $U(t) = \hat{U} \cdot \sin(\omega t + \varphi_u)$, φ_u – Nullphasenwinkel
Phase zum Zeitpunkt $t = 0$

für die Zeiger:

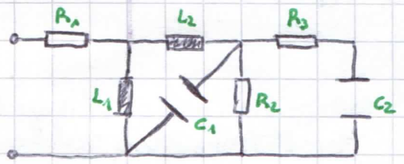
$U = R \cdot I$, $I = \frac{U}{R}$

$U_C = \frac{1}{j\omega C} I = -j \frac{1}{\omega C} I$
↳ $I = U \cdot j\omega C$

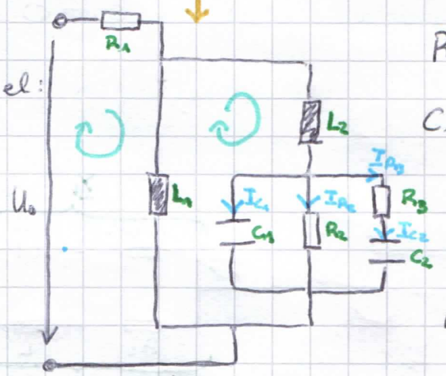
$U_L = j\omega L \cdot I$
↳ $I = \frac{U}{j\omega L} = -j \frac{1}{\omega L} U$



Anwendung auf ein schweres Beispiel:



Ersatzschaltbild ⟹



$R_1 = 1{,}5\,\Omega$, $R_2 = 10\,\Omega$, $R_3 = 3{,}2\,\Omega$

$C_1 = 0{,}2F$, $C_2 = 0{,}1F$

$L_1 = 0{,}25H$, $L_2 = 2H$

$\omega = 1 Hz$
(Wechselstrom)

Lösung für Zeiger:
(Randbedingung)

$I_{R_3} = I_{C_2}$ (Reihenschalt.)

$U_{R_2} = U_{C_1}$ (Parallelschalt.)

$I_{L_2} = I_{C_1} + I_{R_2} + I_{R_3}$

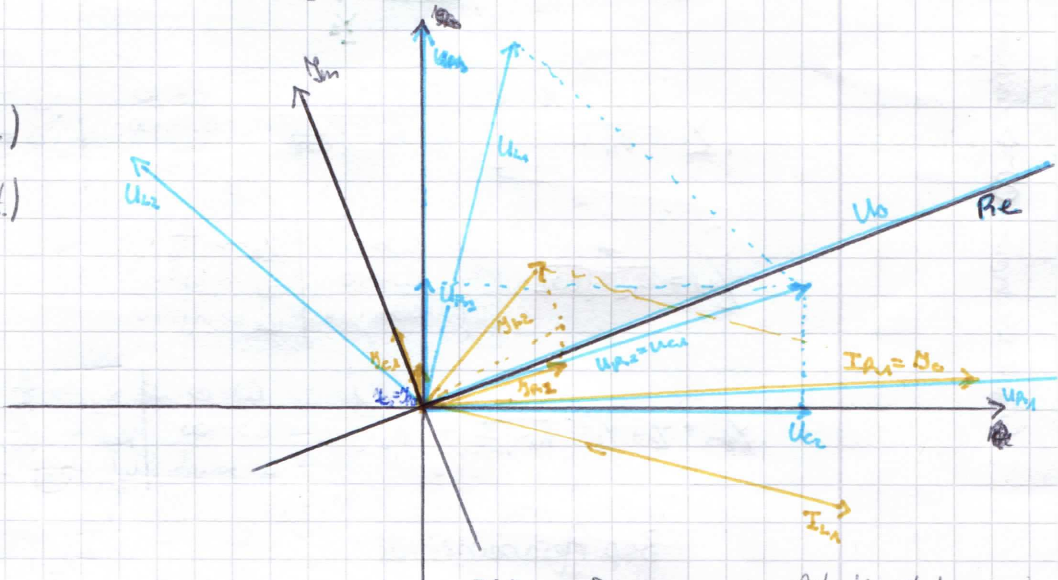
$U_{L_1} = U_{L_2} + U_{R_2}$

$I_{R_1} = I_{L_1} + I_{L_2}$

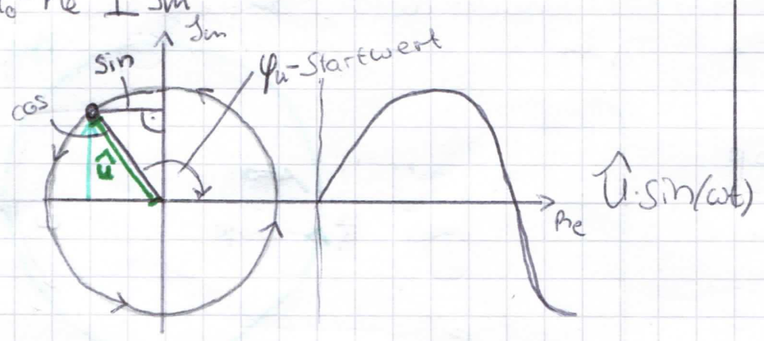
$I_0 = I_{R_1}$

$U_0 = U_{R_1} + U_{L_1}$

$U_0 = Re \perp Im$

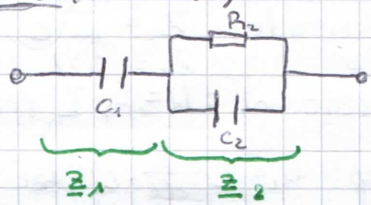


→ wenn Diagramm qualitativ ist, kann man die Phasenverschiebung φ direkt ablesen

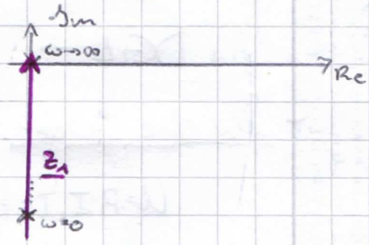


Impedanz & Admittanz (Ortskurve)

1. Schaltung:

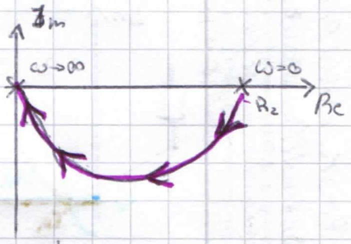


⇒ Impedanzen in Reihe geschaltet kann man addieren $\underline{Z}_{ges} = \underline{Z}_1 + \underline{Z}_2$



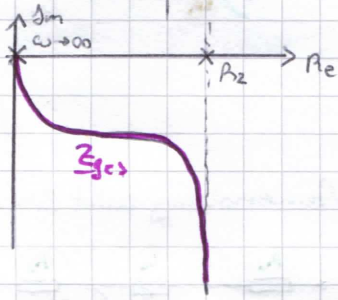
$\underline{Z}_1 = \frac{1}{j\omega C_1} = -j\frac{1}{\omega C}$

$\omega = 0$	$\underline{Z}_1 \to -j\infty$
$\omega \to \infty$	$\underline{Z}_1 \to 0$
ω belieb.	$\underline{Z}_1 \to j$



$\underline{Z}_2 = \frac{1}{\frac{1}{R_2} + j\omega C_2} = \frac{R_2}{1 + j\omega C_2 R_2}$

$\omega = 0$	$R_2 \leftarrow \underline{Z}_2$
ω belieb.	$\underline{Z}_2 \to -j$
$\omega \to \infty$	$\underline{Z}_2 \to 0$

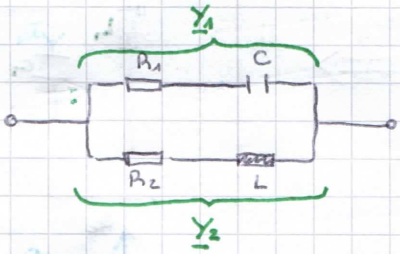


$\underline{Z}_1$ & $\underline{Z}_2$ zusammen:

$\omega \to \infty$	$\underline{Z}_{ges} \to 0$
$\omega \to 0$	$\underline{Z}_{ges} \to -\infty$

Impedanz = Leitwert $\frac{1}{Z}$; $\frac{1}{-j} \xrightarrow{\omega \text{ belieb.}} -j$; $\frac{1}{-j} \xrightarrow{\omega \text{ belieb.}} +j$

2. Schaltung



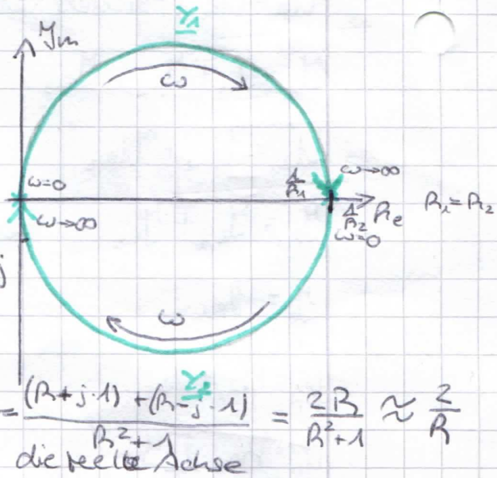
⇒ Admittanzen darf man in Parallelschaltung addieren
$\underline{Y}_{ges} = \underline{Y}_1 + \underline{Y}_2$

$\underline{Y}_1 = \frac{1}{R_1 + \frac{1}{j\omega C}} = \frac{1}{R_1 - j\frac{1}{\omega C}}$

$\omega = 0$	$\underline{Y}_1 \to 0$
$\omega \to \infty$	$\underline{Y}_1 \to \frac{1}{R_1}$
ω belieb.	$\underline{Y}_1 \to +j$

$\underline{Y}_2 = \frac{1}{R_2 + j\omega L}$

$\omega = 0$	$\underline{Y}_2 \to \frac{1}{R_2}$
$\omega \to \infty$	$\underline{Y}_2 \to 0$
ω belieb.	$\underline{Y}_2 \to -j$



$\underline{Y}_{ges} = \underline{Y}_1 + \underline{Y}_2 = \frac{1}{R_1 - j\frac{1}{\omega C}} + \frac{1}{R_2 + j\omega L}$

$\omega = 0$	$\underline{Y}_{ges} \to \frac{1}{R_2}$
$\omega \to \infty$	$\underline{Y}_{ges} \to \frac{1}{R_1}$
ω belieb.	$\underline{Y}_{ges} \to +j\ j - j$

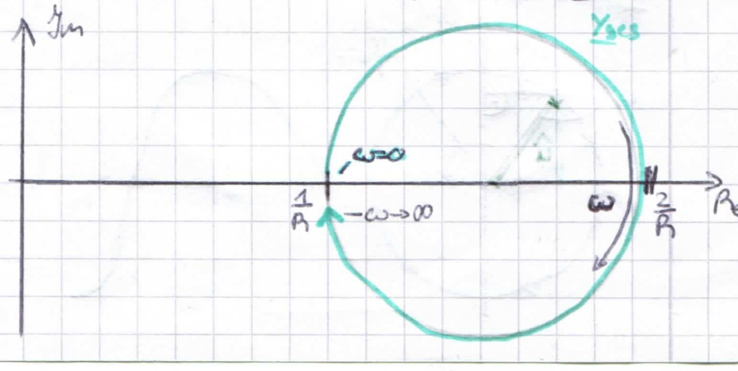
Widerstand → ∞ ⇒ Leitwert → 0

Bsp. Resonanzfall:

$\frac{1}{\omega C} = \omega L \Rightarrow 1 = 1 \Rightarrow \underline{Y}_{ges} = \frac{1}{R - j \cdot 1} + \frac{1}{R + j \cdot 1} = \frac{(R + j \cdot 1) + (R - j \cdot 1)}{R^2 + 1} = \frac{2R}{R^2 + 1} \approx \frac{2}{R}$

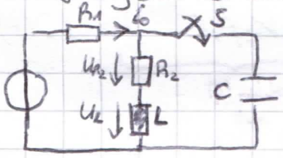
↳ beim Resonanzfall schneidet der Kreis die reelle Achse

Bedeutung:
wenn sich die Blindwiderstände der Spule und des Kondensators aufheben: $\frac{1}{\omega C} = \omega L$
hier: wenn $\omega = 1$ kHz, dann $\frac{1}{\omega L} = \omega L = 1$



Ausgleichsvorgänge

→ Übergang von einem stationären Zustand in einen anderen



$U_L = L \frac{d}{dt} i_L$, $i_C = C \frac{d}{dt} U_C$

$U_0 = 20V, R_1 = 1k\Omega, R_2 = 10\Omega, L = 5H, C = 500mF$

Bei Gleichspannung bietet Spule keinen Widerstand, so fällt sie bei Randbedingung weg

bei Randbedingung betrachtet man das Weg
Wichtig!!!

① Aufstellen der Randbedingungen (Mithilfe von Maschen oder Spannungsteiler)

$i_L(t=0) = \frac{U_0}{R_1+R_2} = 0{,}0198A$ $\hat{=} i_0$

$i_L(t\to\infty) = \frac{U_0}{R_1+R_2}$

$U_C(t=0) = 0V$

$U_C(t\to\infty) = \frac{R_2}{(R_1+R_2)} \cdot U_0$

$U_C = U_{R_2}$

Fall 1: $\delta > \omega_0$

$U_{Ch} = K_1 e^{p_1 t} + K_2 e^{p_2 t}$

$p_1 = -\delta + \sqrt{\delta^2-\omega_0^2}, p_2 = -\delta - \sqrt{\delta^2 - \omega_0^2}$

$i_L(t) = i_{Lh}(t) + i_{Lp}(t), \; i_{Lh}(t) = K_1 e^{p_1 t} + K_2 e^{p_2 t}$

Fall 2: $\delta = \omega_0$

$U_{Ch} = e^{-\delta t}(K_1 + K_2 t)$

$p_1 = p_2 = -\delta$

Fall 3: $\delta < \omega_0$: $U_{Ch} = e^{-\delta t}[K_1 \cos(\omega_d t) + K_2 \sin(\omega_d t)]$

$p_1 = -\delta + \omega_d, p_2 = \delta - \omega_d$ → $\omega_d = \sqrt{\omega_0^2 - \delta^2}$

② DGL aufstellen (für Spannungsquelle: Maschengl., für Stromqu.: Knotengl.)

1. Masche: $U_0 = \boxed{U_{R_1}} + \boxed{U_{R_2}} + \boxed{U_L} \Rightarrow U_0 = \boxed{i_0 \cdot R_1} + \boxed{i_L \cdot R_2} + \boxed{L \cdot \frac{d}{dt} \cdot i_L}$

↳ $i_0 = i_L + i_C \Rightarrow U_0 = (i_L + i_C) R_1 + i_L R_2 + L \frac{d}{dt} i_L$

da i_C unbekannt, also über U_C darstellen : $i_C = C \frac{d}{dt} U_C$

↳ $U_0 = (R_1 + R_2) i_L + C R_1 \frac{d}{dt} U_C + L \frac{d}{dt} i_L$

2. Masche: $U_C = U_{R_2} + U_L$ → einsetzen in U_0

↳ $U_0 = (R_1+R_2) i_L + C R_1 \frac{d}{dt} (U_{R_2} + U_L) + L \frac{d}{dt} i_L$

für: $U_{R_2} = i_L \cdot R_2$ & $U_L = L \cdot \frac{d}{dt} i_L$

⟹ $U_0 = (R_1+R_2) i_L + C R_1 R_2 \frac{d}{dt} i_L + C R_1 L \frac{d^2}{dt^2} i_L + L \frac{d}{dt} i_L$

Normalform: $\frac{d^2}{dt^2} i_L + \underbrace{\frac{C R_1 R_2 + L}{C R_1 L}}_{2\delta} \frac{d}{dt} i_L + \underbrace{\frac{R_1 + R_2}{C R_1 L}}_{\omega_0^2} i_L = \frac{U_0}{C R_1 L}$

③ Fall

$\delta = \frac{C R_1 R_2 + L}{2 C R_1 L} = 1001 \frac{1}{s}$

$\omega_0 = \sqrt{\frac{R_1 + R_2}{C R_1 L}} = 0{,}636 \frac{1}{s}$

$\delta > \omega_0$ → Fall 1.

⟹ $i_{Lh} = K_1 e^{p_1 t} + K_2 e^{p_2 t}$

Weil 1. Fall: $p_1 = -\delta + \sqrt{\delta^2 - \omega_0^2}$, $p_2 = -\delta - \sqrt{\delta^2 - \omega_0^2}$

1. Konst. ⓘ $i_L(t) = i_{Lh}(t) + i_{Lp}(t)$ ⟹ $i_L(t) = \underbrace{K_1 \cdot e^{p_1 t} + K_2 e^{p_2 t}}_{i_{Lh}(t)} + i_{Lp}(t)$

für $t = 0$: $i_L(t=0) = K_1 e^0 + K_2 \cdot e^0 + i_{Lp}(t=0)$

$\frac{U_0}{R_1+R_2} = K_1 + K_2 + \frac{U_0}{R_1+R_2}$ ⟹ $K_1 = -K_2$

2. Konst. ⓘⓘ $U_C(t) = U_{Ch}(t) + U_{Cp}(t)$

aus der **2. Maschengl.** kann man ablesen: $U_{Ch}(t) = U_{R_2 h}(t) + U_{Lh}(t)$

→

die partikulären Lösungen sind die, wo für $t \to \infty$ gilt.

Weil $U_{R_2h}(t) = i_{Lh}(t) \cdot R_2$ & $U_{Lh}(t) = L\frac{d}{dt} i_{Lh}(t)$ gilt:

$$U_{ch} = i_{Lh}(t) \cdot R_2 + L\frac{d}{dt} i_{Lh}(t)$$

$$\hookrightarrow U_c(t) = \underbrace{(K_1 e^{p_1 t} + K_2 e^{p_2 t})}_{i_{Lh}(t)} \cdot R_2 + L \cdot \underbrace{(K_1 p_1 e^{p_1 t} + K_2 p_2 e^{p_2 t})}_{\frac{di_{Lh}(t)}{dt}} + U_{cp}(t)$$

für $t=0$: $U_c(t=0) = (K_1 \cdot 1 + K_2 \cdot 1) \cdot R_2 + L(K_1 p_1 \cdot 1 + K_2 p_2 \cdot 1) + \underbrace{U_{cp}(t=0)}_{\hookrightarrow t \to \infty}$

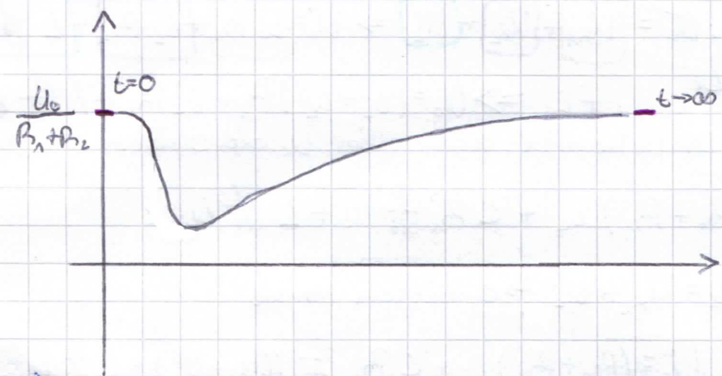
$K_1 = -K_2$: $0 = (-K_2 + K_2) R_2 + L(-K_2 p_1 + K_2 p_2) + \frac{R_2 U_0}{R_1 + R_2}$

$$-L K_2(-p_1 + p_2) = \frac{R_2 U_0}{R_1 + R_2}$$

$$\hookrightarrow K_2 = \frac{R_2 U_0}{(R_1 + R_2)(p_1 - p_2) \cdot L} = -K_1$$

④ Lösung für $i_L(t)$:

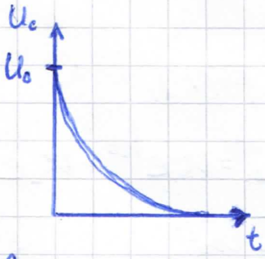
$$i_L(t) = \underbrace{-\frac{R_2 U_0}{(R_1 + R_2)(p_1 - p_2) L}}_{K_1} \cdot e^{p_1 t} + \underbrace{\frac{R_2 U_0}{(R_1 + R_2)(p_1 - p_2) L}}_{K_2} e^{p_2 t} + \frac{U_0}{R_1 + R_2}$$



Theorie

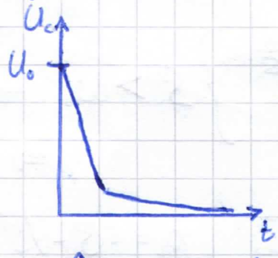
an einem Kondensator kann der Strom springen, aber nicht die Spannung und an einer Spule kann die Spannung springen, aber nicht der Strom

Fall 1:



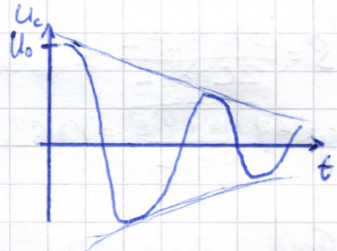
Aperiodischer Fall

Fall 2:



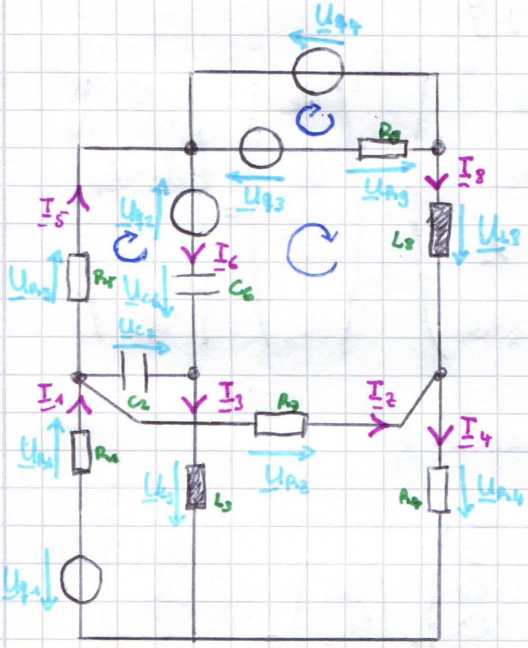
Apperiodischer Grenzfall

Fall 3:



Periodischer Fall

Netzwerkanalyse – Knotenpotentialverfahren



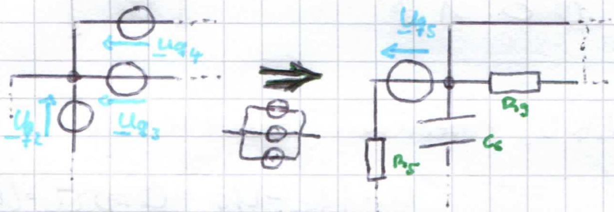
f = 1Hz

$R_1 = 100\,\Omega$, $R_4 = 20\,\Omega$, $R_5 = 5\,\Omega$
$R_7 = 15\,\Omega$, $R_9 = 50\,\Omega$
$C_2 = 5\,mF$, $C_6 = 5\,mF$
$L_3 = 0{,}5\,H$, $L_8 = 0{,}25\,H$
$U_{q1} = 15V$, $U_{q2} = U_{q3} = 10V = U_{q4}$

$U_{k_i} \hat{=} E_i$, $\frac{1}{R_i} = G_i$
→ aus Knoten rausfließende Ströme werden positiv gezählt

Vorgehen:
- Zunächst Knoten wählen (Knotenanzahl ergibt Glch.-Anzahl)
- Spannung bedeutet Potentialunterschied (es gibt ein Bezugspotential – Potentialdiff.)
- Bezugspotential hat meistens 0V ⏚
- Bei dem Verfahren ergibt sich Admittanzmatrix
- Gleichungen werden über Ströme ausgedrückt (anders als beim Maschenstromv. mit Spannungen)

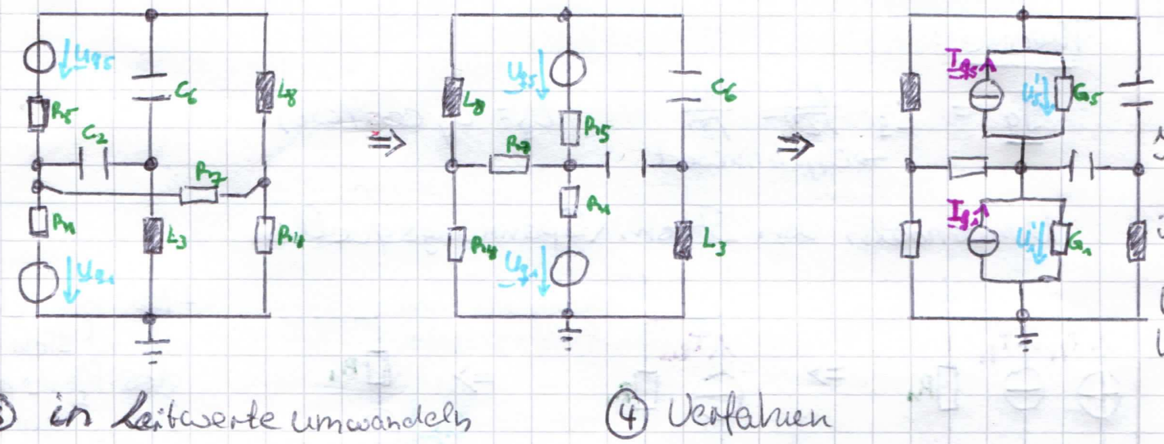
① Quellen zusammenfassen (mit beachten der Potentiale)



$U_{q5} = (U_{q2} = U_{q3} = U_{q4})$

→ R_9 wird kurzgeschlossen

② Umzeichnen



$G_5 = \frac{1}{R_5}$, $G_1 = \frac{1}{R_1}$

$I_{q5} = \frac{U_{q5}}{R_5} = U_{q5} \cdot G_5$

$I_{q1} = U_{q1} \cdot G_1$

$U_5' = U_{q5} - U_5$
$U_1' = U_{q1} - U_1$

③ in Leitwerte umwandeln

$\underline{Y}_8 = \frac{1}{j\omega L_8}$

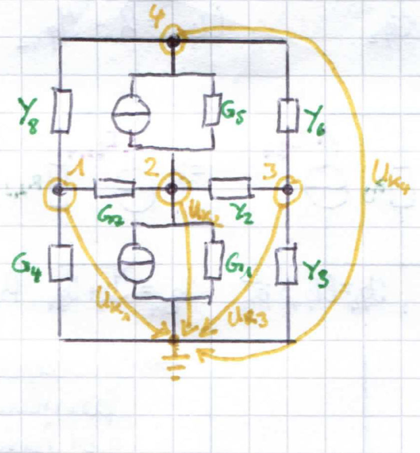
$G_7 = \frac{1}{R_7}$

$\underline{Y}_6 = j\omega C_6$

$G_4 = \frac{1}{R_4}$

$\underline{Y}_3 = \frac{1}{j\omega L_3}$

$\underline{Y}_2 = j\omega C_2$



④ Verfahren

i) $U_{k1} \cdot G_4 + (U_{k1} - U_{k2}) G_7 + (U_{k1} - U_{k4}) \cdot Y_8 = 0$
→ $U_{k1} \cdot (G_4 + G_7 + Y_8) + U_{k2}(-G_7) + U_{k3} \cdot 0 + U_{k4}(-Y_8) = 0$

ii) $(U_{k2} - U_{k1}) \cdot G_7 + (U_{k2} - U_{k3}) Y_2 + U_{k3} \cdot G_1 + (U_{k2} - U_{k4}) \cdot G_5 = I_{q1} - I_{q5}$
→ $U_{k1}(-G_7) + U_{k2}(G_7 + Y_2 + G_1 + G_5) + U_{k3}(-Y_2) + U_{k4}(-G_5) = I_{q1} - I_{q5}$

iii) $U_{k3} \cdot Y_3 + (U_{k3} - U_{k2}) \cdot Y_2 + (U_{k3} - U_{k4}) \cdot Y_6 = 0$
→ $U_{k1} \cdot 0 + U_{k2} \cdot (-Y_2) + U_{k4}(-Y_6) + U_{k3}(Y_3 + Y_2 + Y_6) = 0$

iv) $(U_{k4} - U_{k1}) Y_8 + (U_{k4} - U_{k3}) Y_6 + (U_{k4} - U_{k2}) \cdot G_5 = I_{q5}$
→ $U_{k1}(-Y_8) + U_{k2}(-G_5) + U_{k3}(-Y_6) + U_{k4}(Y_8 + Y_6 + G_5) = I_{q5}$

⑤ Admittanzmatrix

$$Y \cdot \underline{U} = \underline{I} \quad , \underline{U} = inv(Y) \cdot \underline{I}$$

$$\begin{pmatrix} G_4+G_7+Y_8 & -G_7 & 0 & -Y_8 \\ -G_7 & G_7+Y_2+G_1+G_5 & -Y_2 & -G_5 \\ 0 & -Y_2 & Y_3+Y_2+Y_6 & \cancel{-Y_6} \\ -Y_8 & -G_5 & -Y_6 & Y_8+Y_6+G_5 \end{pmatrix} \cdot \begin{pmatrix} U_{k1} \\ U_{k2} \\ U_{k3} \\ U_{k4} \end{pmatrix} = \begin{pmatrix} 0 \\ I_{q1}-I_{q5} \\ 0 \\ I_{q5} \end{pmatrix}$$

$U_{1neu} = U_{k2}$

$U_2 = U_{k2} - U_{k3}$

$U_3 = U_{k3}$

$U_4 = U_{k1}$

$U_{5neu} = U_{k4} - U_{k2}$

$U_6 = U_{k4} - U_{k3}$

$U_7 = U_{k2} - U_{k1}$

$U_8 = U_{k4} - U_{k1}$

Inzidenzenmatrix: B

$$B := \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} U_{k1} \\ U_{k2} \\ U_{k3} \\ U_{k4} \end{pmatrix} = \begin{pmatrix} U_{1neu} \\ U_2 \\ U_3 \\ U_4 \\ U_{5neu} \\ U_6 \\ U_7 \\ U_8 \end{pmatrix}$$

→ U_1 & U_5 berechnen! ⇒ mit Masche:

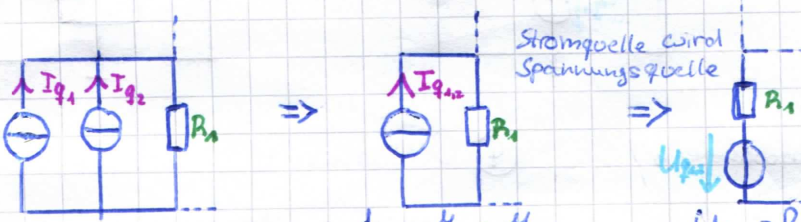
$U_1 + U_{k2} - U_{q1} = 0 \Rightarrow U_1 = U_{q1} - U_{k2}$

Theorie:

$$\underline{I_q} = \hat{i}_q \cdot \frac{1}{\sqrt{2}} = \frac{\hat{I}_q}{\sqrt{2}} \qquad i_q = \hat{i}_q \cdot \cos(\omega t)$$

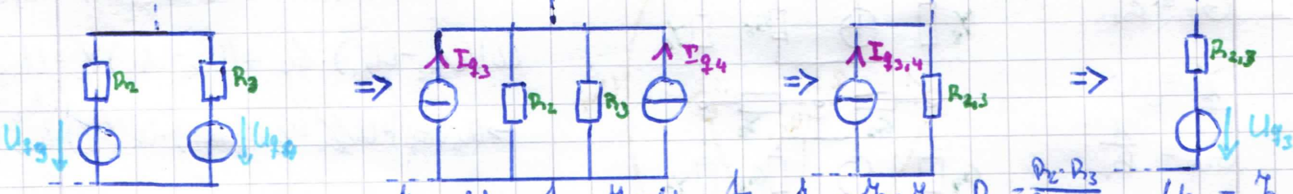
→ Effektivwert

Umwandeln von Strom\Spannungsquellen

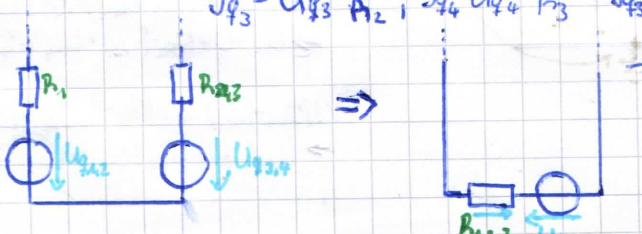


$I_{q1,2} = I_{q1} + I_{q2}$ $\qquad U_{q1,2} = R_1 \cdot I_{q1,2}$

- Bei der Umwandlung dreht sich die Richtung des Pfeils der Quelle um, Spannung zeigt entgegengesetzt zu Strom

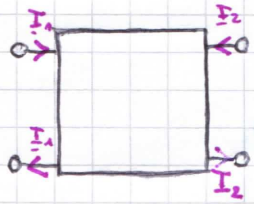


$I_{q3} = U_{q3} \cdot \frac{1}{R_2}$, $I_{q4} = U_{q4} \cdot \frac{1}{R_3}$ $\qquad I_{q3,4} = I_{q3} + I_{q4}$, $R_{2,3} = \frac{R_2 \cdot R_3}{R_2 + R_3}$ $\qquad U_{q3,4} = I_{q3,4} \cdot R_{2,3}$



→ Spannungsquelle mit größerer Spannung "gewinnt" und gibt Richtung des Pfeils vor

Zweitore



- Vierpol, zwei Eingangstor und zwei Ausgangstore
- "Tor", wenn die Torbedingung erfüllt ist, dass der Strom, der in den Anschluss hineinfließt, ohne zeitl. Verzögerung aus dem anderen Anschluss austritt. Bsp: Transformator

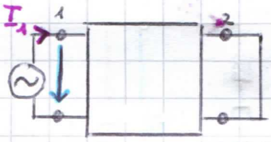
$\begin{pmatrix} \underline{I}_1 \\ \underline{I}_2 \end{pmatrix} = \underline{Y} \cdot \begin{pmatrix} \underline{U}_1 \\ \underline{U}_2 \end{pmatrix} \Rightarrow \underline{Y} = \begin{bmatrix} \underline{Y}_{11} & \underline{Y}_{12} \\ \underline{Y}_{21} & \underline{Y}_{22} \end{bmatrix}$

Admittanzmatrix $\underline{Y}$ (Kurzschluss)

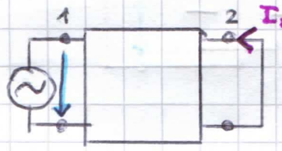
Spannungsquellen

$$\underline{Y}_{11} = \frac{\underline{I}_1}{\underline{U}_1}\bigg|_{U_2=0}$$

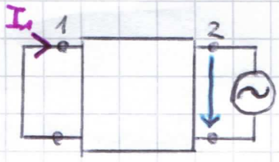
$\underline{I}_1 = \underline{U}_1 \cdot \underline{Y}_{ges}$



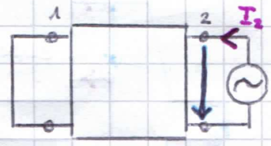
$$\underline{Y}_{21} = \frac{\underline{I}_2}{\underline{U}_1}\bigg|_{U_2=0}$$



$$\underline{Y}_{12} = \frac{\underline{I}_1}{\underline{U}_2}\bigg|_{U_1=0}$$



$$\underline{Y}_{22} = \frac{\underline{I}_2}{\underline{U}_2}\bigg|_{U_1=0}$$

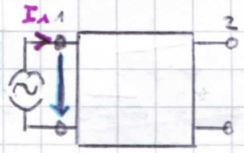


Impedanzmatrix $\underline{Z}$ (Leerlauf)

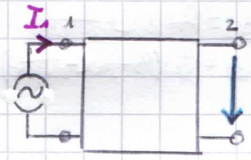
Stromquellen

$\begin{pmatrix} \underline{U}_1 \\ \underline{U}_2 \end{pmatrix} = \underline{Z} \cdot \begin{pmatrix} \underline{I}_1 \\ \underline{I}_2 \end{pmatrix}$

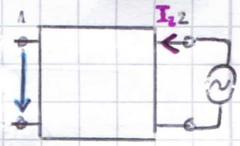
$$\underline{Z}_{11} = \frac{\underline{U}_1}{\underline{I}_1}\bigg|_{I_2=0}$$



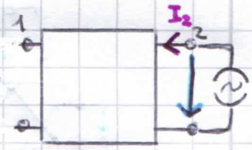
$$\underline{Z}_{21} = \frac{\underline{U}_2}{\underline{I}_1}\bigg|_{I_2=0}$$



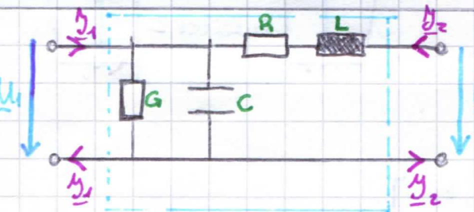
$$\underline{Z}_{12} = \frac{\underline{U}_1}{\underline{I}_2}\bigg|_{I_1=0}$$



$$\underline{Z}_{22} = \frac{\underline{U}_2}{\underline{I}_2}\bigg|_{I_1=0}$$

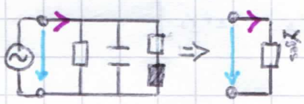


Bsp:



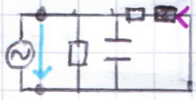
i) $\underline{Y}_{11} = G + j\omega C + \frac{1}{R+j\omega L}$

$\rightarrow \underline{I}_1 = \underline{U}_1 \cdot \underline{Y}_{ges} = \underline{U}_1 \cdot \left(G + j\omega C + \frac{1}{R+j\omega L}\right)$



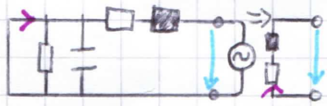
ii) $\underline{Y}_{21} = -\frac{1}{R+j\omega L}$

$\rightarrow \underline{I}_2 = \underline{U}_1 \cdot \left(-\frac{1}{R+j\omega L}\right)$



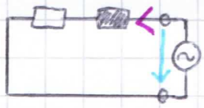
iii) $\underline{Y}_{12} = -\frac{1}{R+j\omega L}$

$\rightarrow \underline{I}_1 = \underline{U}_2 \cdot \left(-\frac{1}{R+j\omega L}\right)$



iv) $\underline{Y}_{22} = \frac{1}{R+j\omega L}$

$\rightarrow \underline{I}_2 = \underline{U}_2 \cdot \frac{1}{R+j\omega L}$



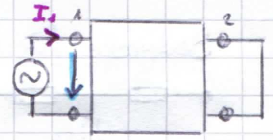
Admittanzen

$$\underline{Y} = \begin{bmatrix} G + j\omega L + \frac{1}{R+j\omega L} & -\frac{1}{R+j\omega L} \\ -\frac{1}{R+j\omega L} & \frac{1}{R+j\omega L} \end{bmatrix}$$

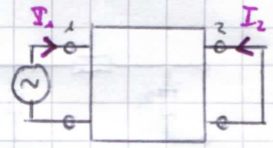
Reihenparallelmatrix $\underline{H}$ (Hybridmatrix)

$\begin{pmatrix} \underline{U}_1 \\ \underline{I}_2 \end{pmatrix} = \underline{H} \cdot \begin{pmatrix} \underline{I}_1 \\ \underline{U}_2 \end{pmatrix}$

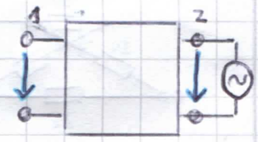
$$\underline{H}_{11} = \frac{\underline{U}_1}{\underline{I}_1}\bigg|_{U_2=0}$$



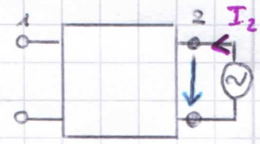
$$\underline{H}_{21} = \frac{\underline{I}_2}{\underline{I}_1}\bigg|_{U_2=0}$$



$$\underline{H}_{12} = \frac{\underline{U}_1}{\underline{U}_2}\bigg|_{I_1=0}$$



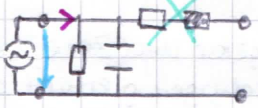
$$\underline{H}_{22} = \frac{\underline{I}_2}{\underline{U}_2}\bigg|_{I_1=0}$$



Selbes Bsp. mit Impedanzen

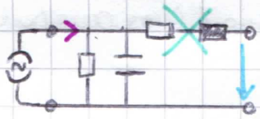
i) $\underline{Z}_{11} = \frac{1}{G+j\omega C}$

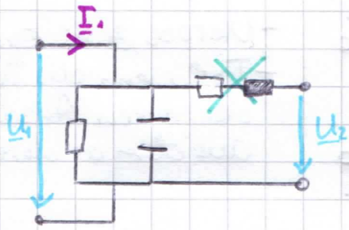
$\underline{U}_1 = \underline{I}_1 \cdot \frac{1}{G+j\omega C}$



ii) $\underline{Z}_{21} = \frac{1}{G+j\omega C}$

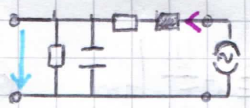
$\underline{U}_2 = \underline{I}_1 \cdot \frac{1}{G+j\omega C}$





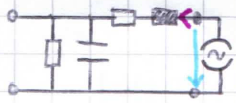
iii) $\underline{Z}_{12} = \frac{1}{G+j\omega C}$

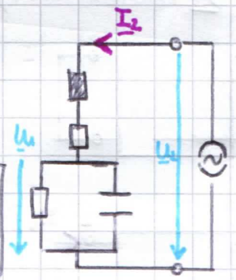
$\underline{U}_1 = \underline{I}_2 \cdot \frac{1}{G+j\omega C}$



iv) $\underline{Z}_{22} = R+j\omega L+\frac{1}{G+j\omega C}$

$\underline{U}_2 = \underline{I}_2 \cdot (R+j\omega L+\frac{1}{G+j\omega C})$

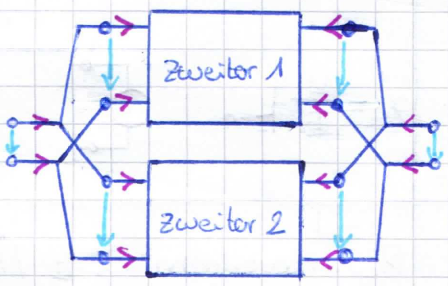




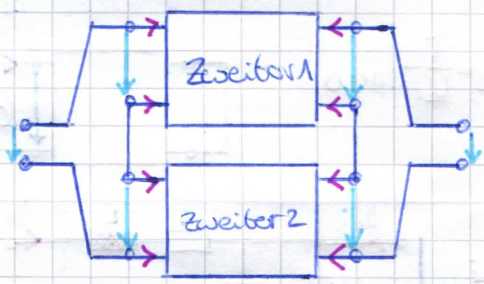
Theorie

→ Schaltung:

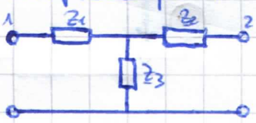
1) Parallelschaltung:



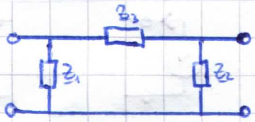
2) Reihenschaltung



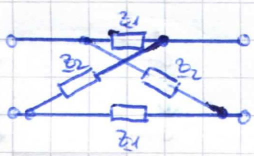
Beispiele für Matrizen:



$$\underline{Z} := \begin{bmatrix} Z_1+Z_3 & Z_3 \\ Z_3 & Z_2+Z_3 \end{bmatrix}$$

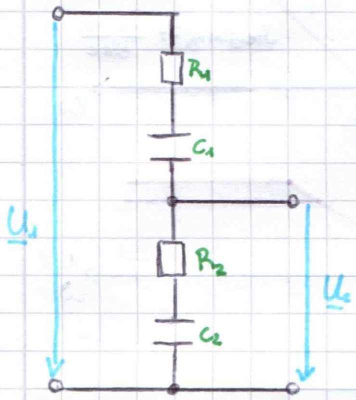


$$\underline{Y} := \begin{bmatrix} Y_1+Y_3 & -Y_3 \\ -Y_3 & Y_2+Y_3 \end{bmatrix}$$



$$\underline{Z} := \frac{1}{2} \begin{bmatrix} Z_2+Z_1 & Z_2-Z_1 \\ Z_2-Z_1 & Z_2+Z_1 \end{bmatrix} \qquad \underline{Y} := \frac{1}{2} \begin{bmatrix} Y_2+Y_1 & Y_2-Y_1 \\ Y_2-Y_1 & Y_2+Y_1 \end{bmatrix}$$

Bodediagramm & Frequenzgänge



$$\underline{V} = \frac{U_2}{U_1} = \frac{(R_2 + \frac{1}{j\omega C_2})\cancel{\mathcal{Y}}}{(R_1+R_2+\frac{1}{j\omega C_1}+\frac{1}{j\omega C_2})\cancel{\mathcal{Y}}} \quad \Big| \cdot \left(\frac{j\omega C_2}{j\omega C_2}\right)$$

$$= \frac{R_2 j\omega C_2 + 1}{(R_1+R_2)j\omega C_2 + \cancel{\frac{j\omega C_2}{j\omega C_1}} + 1} \quad \Big| \quad 1 = \frac{C_1}{C_1}$$

$$= \frac{j\omega C_2 R_2 + 1}{j\omega C_2(R_1+R_2) + \frac{C_2+C_1}{C_1}} \quad \Big| \cdot \left(\frac{\frac{C_1}{C_2+C_1}}{\frac{C_1}{C_2+C_1}}\right)$$

$$= \underbrace{\left(\frac{C_1}{C_2+C_1}\right)}_{K} \cdot \left(j\omega \underbrace{C_2 R_2}_{\tau_2} + 1\right) \cdot \frac{1}{j\omega \underbrace{C_2\left(\frac{(R_1+R_2)C_1}{C_1+C_2}\right)}_{\tau_1} + 1}$$

$\hookrightarrow \underline{V_2} = j\omega\tau_2 + 1 \qquad \underline{V_1} = \frac{1}{j\omega\tau_1 + 1}$

im Skript: $\underline{H}(j\omega) = \frac{\underline{U_A}}{\underline{U_E}} = \frac{U_A \cdot e^{j\varphi_A}}{U_E \cdot e^{j\varphi_E}}$

$\boxed{\underline{V} = \frac{\underline{U_2}}{\underline{U_1}}}$ $\Rightarrow$ es gibt höchstens 1 und mindestens 0

$P = U \cdot I,\ U = R \cdot I \Rightarrow I = \frac{U}{R}$

$\hookrightarrow P = \frac{U^2}{I}$

$\tau = [s]$

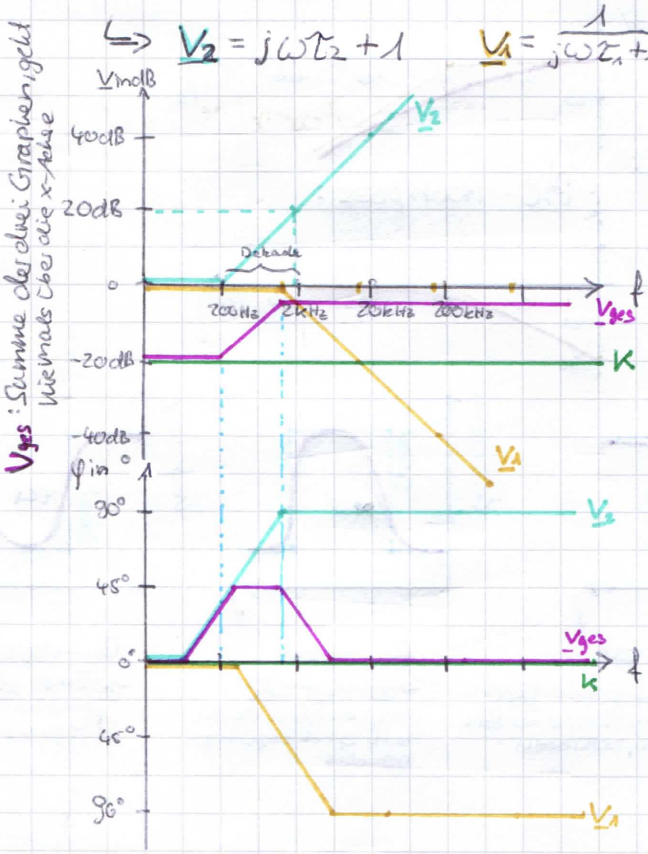
$K = \frac{C_1}{C_1+C_2} = \frac{1nF}{10nF+1nF} = 0{,}09 \rightarrow 20\cdot\log(K) = 20{,}8\,dB$

$\tau_2 = R_2 \cdot C_2 = 80k\Omega \cdot 10nF = 0{,}8\,ms$

$\tau_1 = \frac{C_1 \cdot C_2(R_1+R_2)}{C_2+C_1} = 91\,\mu s$

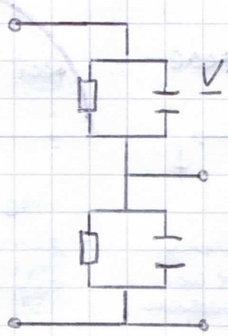
$\frac{1}{\tau_1} = \omega_1 = 11000\,\frac{1}{s} \rightarrow f_1 = \frac{\omega_1}{2\pi} = 1{,}75\,kHz$

$\frac{1}{\tau_2} = \omega_2 = 1250\,\frac{1}{s} \rightarrow f_2 = \frac{\omega_2}{2\pi} = 199\,Hz$



$\Rightarrow$ Hochpass

Bsp 2: Tastkopf



$$\underline{V} = \frac{\frac{1}{\frac{1}{R_1}+j\omega C_2}}{\frac{1}{\frac{1}{R_1}+j\omega C_1} + \frac{1}{\frac{1}{R_2}+j\omega C_2}}$$

$$= \frac{\frac{R_2}{1+j\omega C_2 R_2}}{\frac{R_1}{1+j\omega R_1 C_1} + \frac{R_2}{1+j\omega C_2 R_2}}$$

$[\tau_1 = R_1 C_1,\ \tau_2 = R_2 C_2]$

$$= \frac{\frac{R_2}{j\omega\tau_2+1}}{\frac{R_1(1+j\omega\tau_2)+R_2(1+j\omega\tau_1)}{(1+j\omega\tau_1)\cdot(1+j\omega\tau_2)}}$$

$$\frac{\frac{R_2}{R_1+R_2}(1+j\omega\tau_1)}{1+j\omega\cdot\underbrace{\frac{\tau_2 R_1+\tau_1 R_2}{R_1+R_2}}_{\tau_3}} = \underbrace{\frac{R_2}{R_1+R_2}}_{K} \cdot \underbrace{(1+j\omega\tau_1)}_{V_1} \cdot \underbrace{\frac{1}{1+j\omega\tau_3}}_{V_2}$$

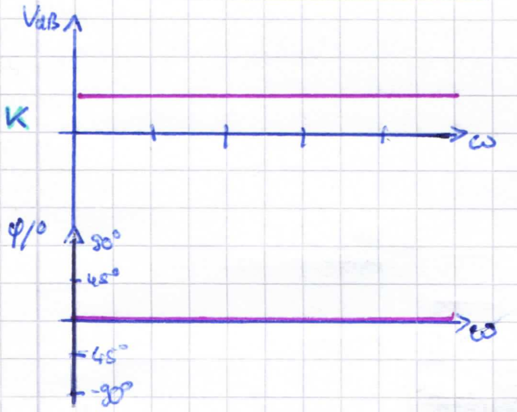
$\Leftarrow$

$K = \frac{R_2}{R_1+R_2} = \frac{1k\Omega}{10k\Omega+1k\Omega} = \frac{1}{11} = 0{,}09$

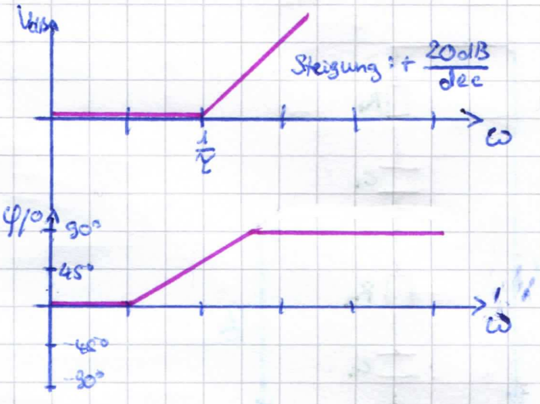
$20\log(K) = -20{,}8\,dB$

Theorie (Formelsammlung)

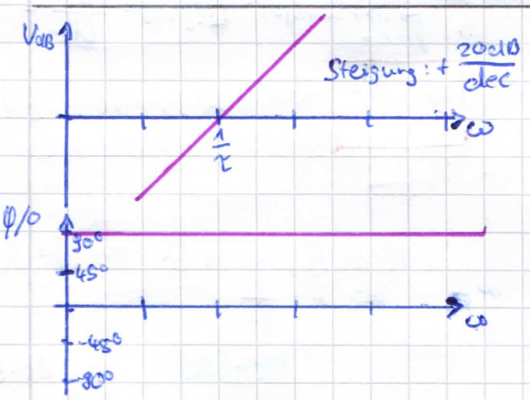
$\underline{V} = K$



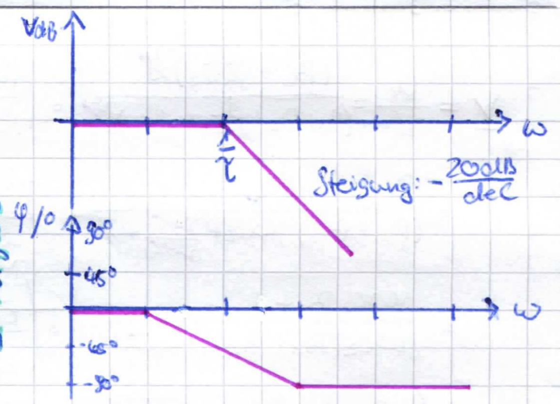
$\underline{V} = 1 + j\omega\tau$



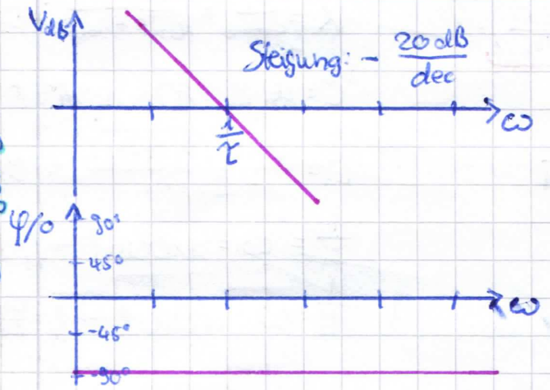
$\underline{V} = j\omega\tau$



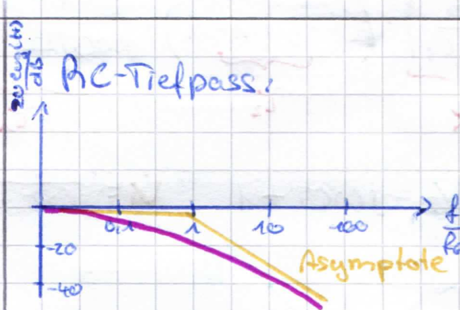
$\underline{V} = \frac{1}{1 + j\omega\tau}$



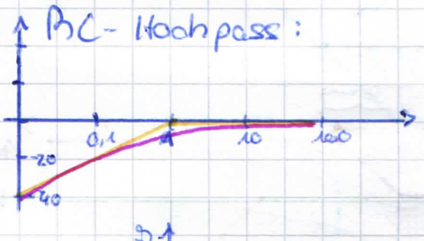
$\underline{V} = \frac{1}{j\omega\tau}$



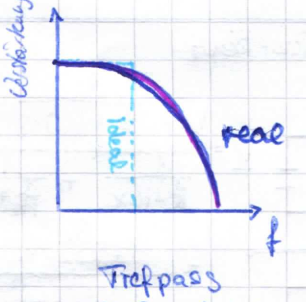
RC-Tiefpass:



RC-Hochpass:

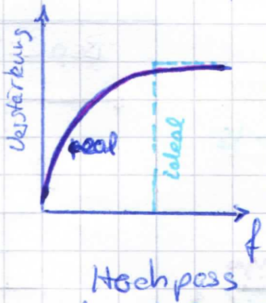


Allgemein: (Filter)



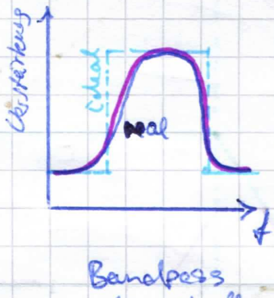
Tiefpass

→ dämpft Signale hoher Frequenzen, lässt aber tiefe Frequenzen passieren



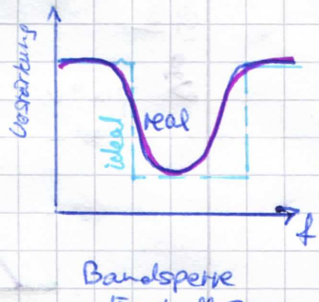
Hochpass

→ dämpft Signale tiefer Frequenzen, lässt aber hohe Frequenzen passieren



Bandpass

→ dämpft alle Frequenzen außerhalb eines Frequenzbandes



Bandsperre

→ dämpft alle Frequenzen innerhalb eines Frequenzbandes

$\tau = RC$ als Zeitkonstante

$\omega_G = \frac{1}{\tau} = \frac{1}{RC}$ als Grenzfrequenz

$\varphi_H = -\arctan(\omega\tau) = -\arctan\left(\frac{\omega}{\omega_G}\right)$

Zur Berechnung

Betragsgang: $|\underline{H}| = \sqrt{Re(H)^2 + Im(H)^2}$ mit $|\underline{H}|_{dB} = 20 \log(|H|)$ (im Beispiel ist $\underline{H} = \underline{V}$)

Phasengang: $\arg(\underline{H}) = \arctan\left(\frac{Im(H)}{Re(H)}\right)$

→ wenn dB-Zahl positiv ist, dann ist $U_A > U_E$
↳ Verstärkung

→ wenn dB-Zahl negativ ist, dann ist $U_A < U_E$
↳ Dämpfung