

# Hybrid Systems Gedächtnisprotokoll

Juli 2022

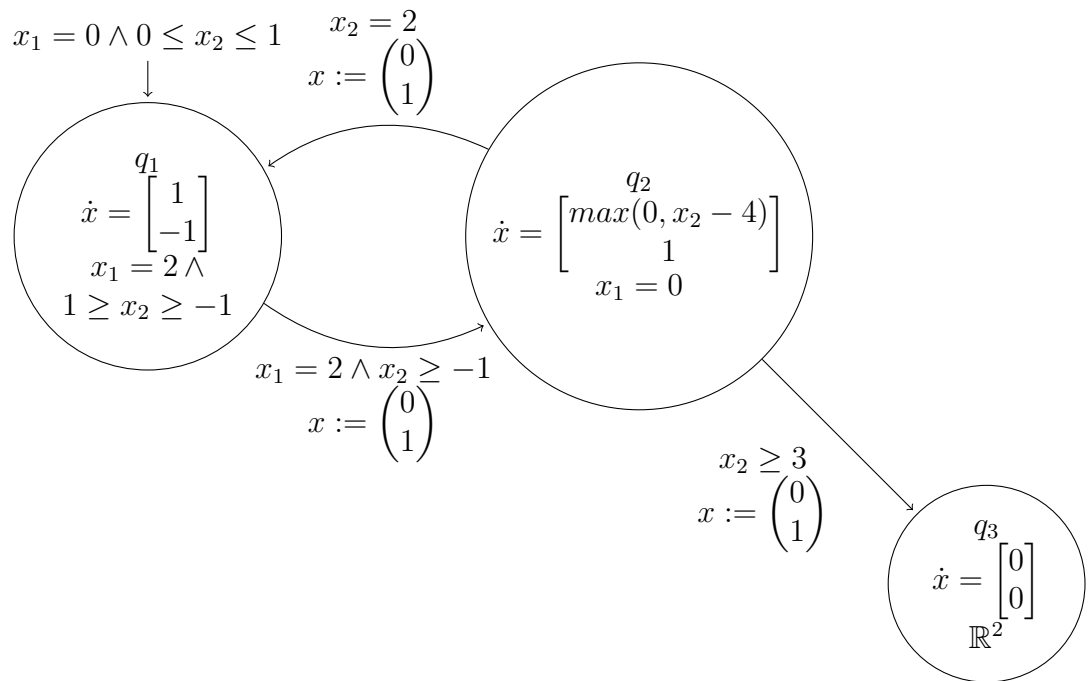
## Q1 (7 pts)

Consider the relation  $R \supseteq \mathbb{R} \times \mathbb{R}$  and  $k \in \mathbb{Z}$ .

$$R = \{(a, b) \in \mathbb{R} \times \mathbb{R} \mid \exists k \in \mathbb{Z} : a = 5^k b\}$$

Is R an equivalence relation? Justify.

## Q2 (33 pts)



a) (8 pts) Decide if swichting is:

- necessary and possible
- necessary, but not possible
- not necessary and possible
- neither possible nor necessary

For  $(q_1, \begin{bmatrix} 1 \\ -1 \end{bmatrix}), (q_2, \begin{bmatrix} 0 \\ 2 \end{bmatrix}), (q_2, \begin{bmatrix} 0 \\ 3 \end{bmatrix}), (q_2, \begin{bmatrix} 0 \\ 4 \end{bmatrix})$ .

b) (8 pts) Is HA deterministic? Justify.

c) (9 pts) Is HA blocking? Justify.

d) (8 pts) Provide an execution of HA from  $t=0$  to  $t=7$ . With initial condition  $(q_1, \begin{bmatrix} 0 \\ 1 \end{bmatrix})$ . Show the evolution of discrete and continuous states and specify the corresponding hybrid time set.

### Q3 (19 pts)

Is the switched linear system  $S_\sigma$  GUAS? Justify.

a) N=2  $A_1 = \begin{bmatrix} -1 & 5 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$   $A_2 = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -5 \end{bmatrix}$

b) N=10  $A_k = (-5)^k, \quad k \in \{1, 2, 3, \dots, 10\}$

c) N=2  $A_1 = \begin{bmatrix} -3 & 2 \\ 0 & -2 \end{bmatrix}$   $A_2 = \begin{bmatrix} 1 & 3 \\ -1 & -2 \end{bmatrix}$

### Q4 (8 pts)

Consider the switched linear system  $S_\sigma : \dot{x}(t) = A_{\sigma(t)}x(t)$  with  $\sigma : \mathbb{R}^+ \rightarrow \{1, 2, 3, 4, 5\}$ . Answer the question; no justification needed.

- if  $\forall \sigma : A_{ij} = A_{ji}$  the system is GUAS.

- if  $S_\sigma$  is GUAS  $\Rightarrow A_{ij} = A_{ji} \forall i, j \in \{1, \dots, 5\}$
- if there is a JQLF for every  $A_\sigma : S_\sigma$  is GUES
- if  $S_\sigma$  is GUAS there is a JQLF for every  $A_\sigma$

### Q5 (22 pts)

Consider the system  $\Sigma = (\mathbb{N}_0, W, \mathcal{B})$

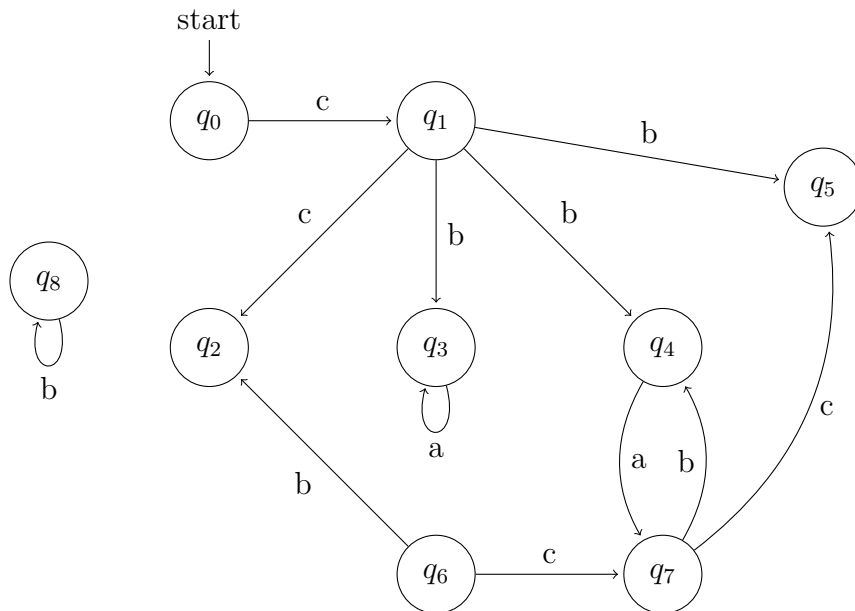
with  $W = (a, b)$ ,  $\mathcal{B} = \{aaaaaaaa\dots, bbaaaaa\dots, bbbbaaaa\dots, bbbbbbaaaa\dots, \dots\}$

- Provide a state machine graphically
- Time invariant / strictly time invariant?
- Is  $\Sigma$  2-complete ?

### Q6 (11 pts)

Consider the state machine P.

P :



- a) (3pts) Provide reachable part  $P'$ .
- b) (3pts) Provide a nonblocking  $P''$  with the same behavior as  $P'$ .
- c) (5pts) Is  $P''$  past-induced? Justify.