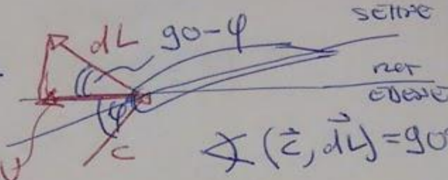
@ Gordon

t_{BETZ} Herleitung

Betz

i) $dP_{AERODYNAMISCH} = dP_{BETZ}$ IST DEIN ANSATZ

$$dP_{AERO} = dU_{TOT} \cdot r \cdot \Omega \; ; \; dU_{TOT} = dL \sin\varphi \cdot z$$



$\sphericalangle(\vec{c}, \vec{dL}) = 90°$

$\underbrace{F \cdot r}$

$P = \text{MOMENT} \cdot \Omega$

$P = M\Omega$

WEIL DU JA „z" ROTORBLÄTTER HAST, JEDES MIT EIN dU

$$dU = dL \cos(90-\varphi) = dL \sin\varphi$$

$$\underline{dU_{TOT} = dU \cdot z}$$

ii) $dP_{AERO} = dP_{BETZ}$

$$dL \sin\varphi \; z \cdot r \cdot \Omega = dP_{BETZ}$$

$$\frac{1}{\cancel{2}}\cancel{\rho} c^2 t\, dr\, c_L \cdot \frac{2}{3}\frac{v_1}{c} \cdot z \cdot r\Omega = \frac{1}{\cancel{2}}\cancel{\rho} v_1^3\, dA\, c_P$$

$c_{P,OPT}$ NACH BETZ

$$c^{\cancel{2}}\, t\, \cancel{dr} \cdot c_L \cdot \frac{2}{3}\frac{v_1}{\cancel{c}} \cdot z\, \cancel{r}\Omega = v_1^{\cancel{3}\,2} \cdot 2\pi \cancel{r\, dr} \cdot \frac{16}{27}$$

$$c\, t\, c_L\, \frac{\cancel{2}}{\cancel{3}}\, z\, \Omega = v_1^2 \cdot 2\pi\, \frac{16}{27}\, \frac{8}{9}$$

$$\cancel{v_1}\sqrt{\frac{4}{9} + \left(\frac{\Omega r}{v_1}\right)^2} \cdot t\, c_L\, z\, \Omega = v_1^{\cancel{2}}\, 2\pi\, \frac{8}{9}$$

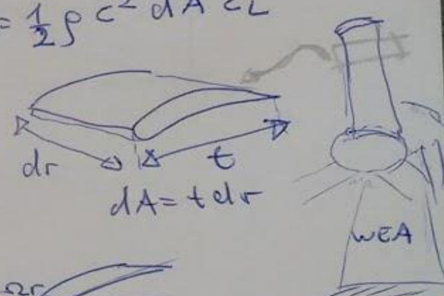
$$\lambda_D = \frac{\Omega R}{v_1} \Rightarrow v_1 = \frac{\Omega R}{\lambda_D}$$

HIER $\lambda = \lambda_D$ WEIL ICH IM AUSLEGUNGSPUNKT BIN (BETZ ... $c_P = 16/27$)

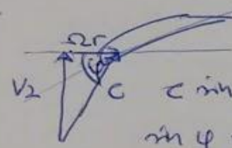
$$\sqrt{\frac{4}{9} + \left(\frac{\Omega r}{\frac{\Omega R}{\lambda_D}}\right)^2} \cdot t\; c_L\, z\, \cancel{\Omega} = \frac{\cancel{\Omega} R}{\lambda_D}\, 2\pi\, \frac{8}{9}$$

$$\Rightarrow \boxed{t(r) = \frac{2\pi R}{\lambda_D}\, \frac{8}{9} \cdot \frac{1}{c_L z} \cdot \frac{1}{\sqrt{\frac{4}{9} + \left(\frac{r}{R}\right)^2 \lambda_D^2}}}$$

$$dL = \frac{1}{2}\rho c^2\, dA\, c_L$$



$$dA = t\, dr$$



$$c \sin\varphi = v_2$$

$$\sin\varphi = \frac{v_2}{c} = \frac{2}{3}\frac{v_1}{c}$$

WEIL NACH BETZ $v_2 = \frac{2}{3} v_1$ OPTIMUM

$$dA = 2\pi r\, dr$$

$$c = \sqrt{v_2^2 + (\Omega r)^2} \quad \text{PYTHAGORAS}$$

$$= \sqrt{\left(\frac{2}{3}v_1\right)^2 + (\Omega r)^2} =$$

$$= \sqrt{\frac{4}{9}v_1^2 + (\Omega r)^2 \frac{v_1^2}{v_1^2}} =$$

$$\boxed{= v_1\sqrt{\frac{4}{9} + \left(\frac{\Omega r}{v_1}\right)^2} = c}$$