mdl. Ana I Prüfung 24/09/18

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Wunschthema: Integrale

$$\int_2^{\infty} \frac{4x^2}{x^4-1}\,dx = \lim_{\beta \to \infty} \int_2^{\beta} \frac{4x^2}{x^4-1}\,dx \overset{PBZ}{=}$$

Was für Fkttyp:

rationale Fkt +

uneigentliches Integral

NR PBZ:

$$\frac{4x^2}{x^4-1} = \frac{4x^2}{(x-1)(x+1)(x+i)(x-i)}$$

Fundamentalsatz der Algebra

Linearfaktorzerlegung

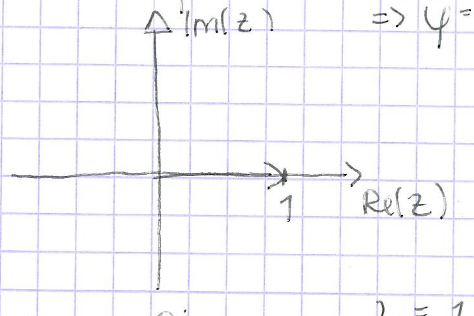
$x^4 - 1 = 0$

$x^4 = 1$

$\to z^4 = 1$

De Moivre

$r = 1$ $\quad \varphi =$ zeichnerisch $\Rightarrow \varphi = 0$




$z^4 = 1e^{0i} = 1$

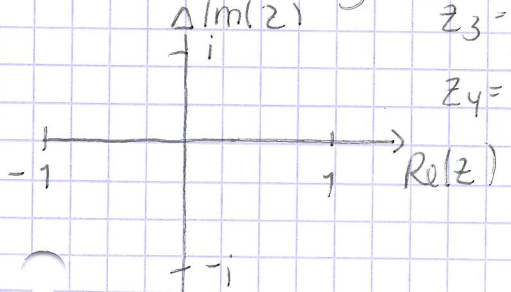
$z_1 = 1$

$z_2 = -1$

$z_3 = i$

$z_4 = -i$

zeichnerische Lsg.




komplexe Zerlegung

$$= \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+i} + \frac{D}{x-i}$$

reelle Zerlegung

$$= \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{(x+i)(x-i)}$$

$$= \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$$

für A und B Zuhaltemethode:

$A = \frac{4}{4} = 1 \qquad B = \frac{4}{-4} = -1$

für C und D Koeffizientenvgl.:

Zahlen gegeben vom Prof

$C = 0 \qquad D = 2$

$$\Rightarrow \frac{1}{x-1} + \frac{-1}{x+1} + \frac{2}{x^2+1}$$

$$\Rightarrow \lim_{\beta \to \infty} \int_2^{\beta} \frac{1}{x-1} - \frac{1}{x+1} + \frac{2}{x^2+1}\,dx$$

$$= \lim_{\beta \to \infty} \ln|x-1| - \ln|x+1| + 2\arctan(x) \Big|_2^{\beta}$$

$$\lim_{\beta\to\infty} \ln|x-1| - \ln|x+1| + 2\arctan(x) \Big|_2^{\beta}$$

$$= \lim_{\beta\to\infty} \ln|\beta-1| - \ln|\beta+1| + 2\arctan(\beta) - \Big(\overbrace{\ln|2-1|}^{0} - \ln|2+1| + 2\arctan(2)\Big)$$

einzelne Betrachtung:

$$\lim_{\beta\to\infty} \ln|\beta-1| - \ln|\beta+1| = \lim_{\beta\to\infty} \ln\left(\frac{\beta-1}{\beta+1}\right) \underset{A}{=} \ln\left(\lim_{\beta\to\infty} \frac{\beta-1}{\beta+1}\right)$$

Betragsstriche weg, da positiv

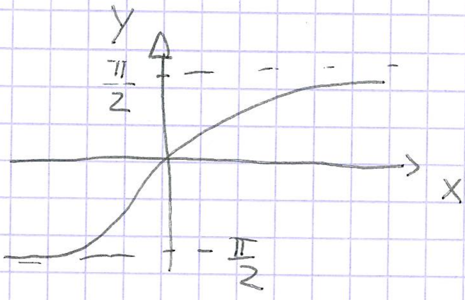
A: ln ist stetig

$$= \ln(1) = 0$$

NR 2:

$$\lim_{\beta\to\infty} \frac{\overbrace{\beta-1}^{\infty}}{\underbrace{\beta+1}_{\infty}} \overset{L'H}{=} \lim_{\beta\to\infty} \frac{1}{1} = 1$$

$\lim_{\beta\to\infty} \arctan(\beta)$ zeichnerische Lsg




$$= \lim_{\beta\to\infty} \arctan(\beta) = \frac{\pi}{2}$$

=> Integrale existiert, Rest war uninteressant

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$f(x) = x e^{-x}$ 15. Ableitung ? und punkt von 0,1 ↲ $x_0 = 0$

Ableitung n-ter Ordnung

Taylorpolynom

$$f'(x) = e^{-x} + (-x e^{-x}) = -e^{-x}(x-1)$$

$$f''(x) = e^{-x}(x-1) + (-e^{-x}) = e^{-x}(x-2)$$

$$f'''(x) = -e^{-x}(x-2) + e^{-x} = -e^{-x}(x-3)$$

Vermutung:

$$f^{(n)}(x) = (e^{-x})(-1)^n(x-n)$$

$$f^{(n)}(x) = (-1)^n e^{-x}(x-n)$$

=> vollständige Induktion

IA: $n=0 \to 0=0$ w.A.

IV: ...

IB $\to n+1$ $f^{(n+1)}(x) = (-1)^{n+1} e^{-x}(x-(n+1))$

IS: $f^{(n+1)}(x) = (f^{(n)}(x))' \overset{IV}{=} \dots$ dann fertig!