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Midterm written exam in course "Discrete Event Systems"

26.02.2021

Maximum score: 40 points

Question 1 (15 points)

Consider the Petri net shown in Figure 1.

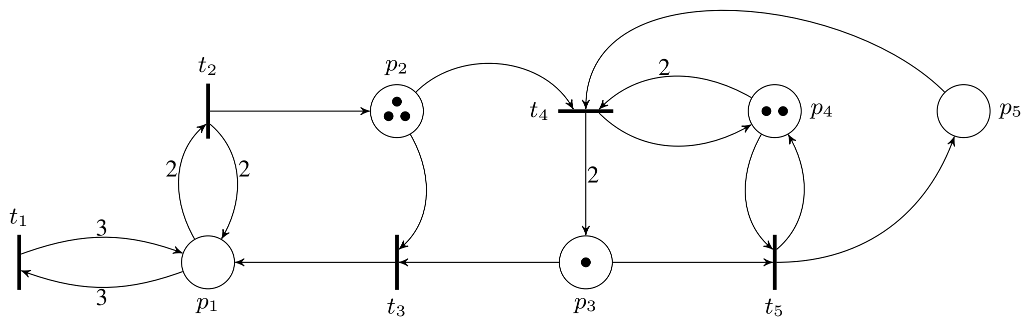

Figure 1: Petri net for Question 1.

a) (6 points)
Construct the coverability tree for the given Petri net.

b) (7,5 points)
Determine whether each transition in the Petri net is

- dead;
- L1-live and not L3-live;
- L3-live and not live; or
- live.

Please choose one of the options above for each transition. Note that exactly one of the options is correct for any given transition.

c) (1 point)
Is the Petri net persistent? Justify your answer.

d) (0,5 point)
Is the Petri net bounded? Justify your answer.

Question 2 (8 points)

A plant to be controlled is modeled by the Petri net shown in Figure 2. A robot with single capacity takes unprocessed workpieces (transition t_1) and loads them into machines M_1 and M_2 (t_2 and t_4, respectively) to be processed. After processing, a second robot, also with single capacity, collects the workpieces from M_1 and M_2 (t_3 and t_5, respectively) and deposits them onto an output conveyor belt (t_6). As represented by the initial marking in Figure 2, initially the second robot holds a workpiece, machine M_2 is processing two workpieces, whereas M_1 and the first robot are empty. The following specifications are given:

- the two robots cannot both hold workpieces at the same time;
- M_1 can never be processing more workpieces than M_2;
- M_1 and M_2 combined cannot process more than 9 workpieces at a time.

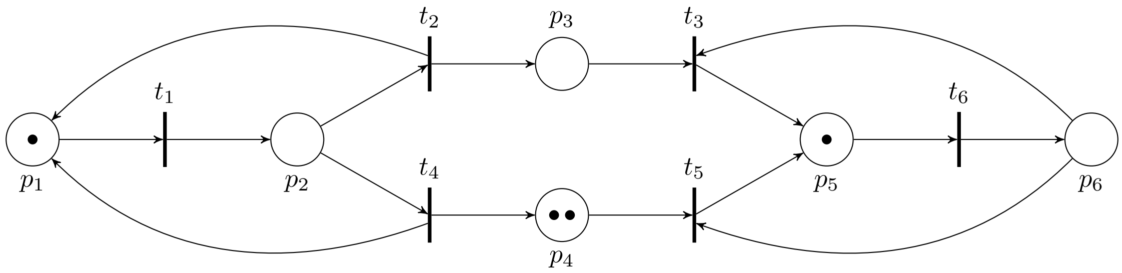

Figure 2: Plant model for Question 2.

a) (6 points)
Write the specifications in the form $\Gamma x(k) \leq b$ and compute the corresponding least restrictive (ideal) controller. Provide the controller's incidence matrix, A_c, and initial marking, x_c^0.

b) (2 points)
Assume, now, that transitions t_4 and t_6 are observable but not controllable, whereas all other transitions are both controllable and observable. Are the given specifications ideally enforceable? Justify your answer.

Question 3 (4 points)

Determine the elements $(M^4)_{41}$ and $(M^3)_{44}$ for the max-plus matrix M given by

$$M = \begin{bmatrix} \varepsilon & -1 & -5 & 6 \\ 4 & 2 & 3 & \varepsilon \\ \varepsilon & e & 6 & \varepsilon \\ 1 & \varepsilon & \varepsilon & \varepsilon \end{bmatrix}.$$

Question 4 (13 points)

Consider the timed event graph provided in Figure 3.

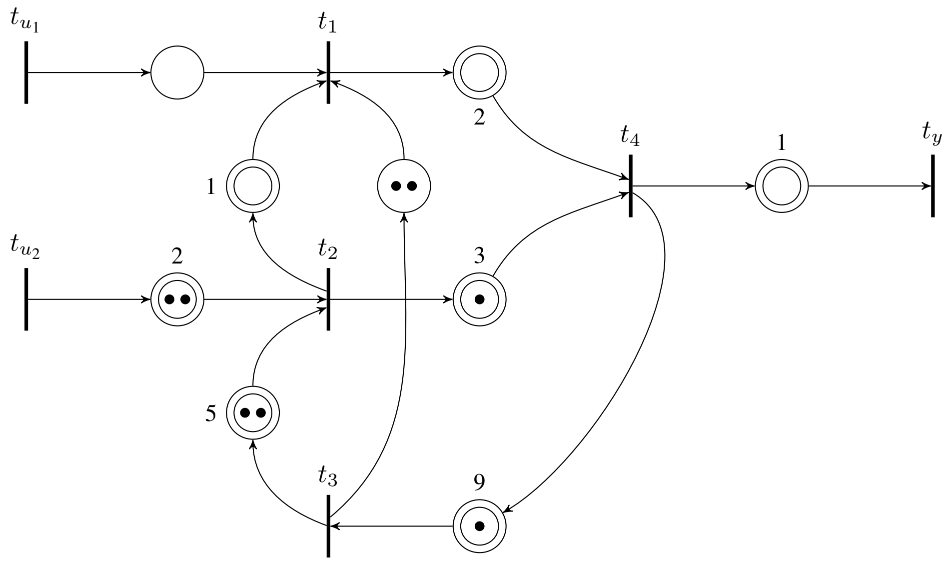

Figure 3: Timed event graph for Question 4.

a) (6 points)
Obtain max-plus equations describing the firing times of the transitions, in the form

$$x(k+1) = \bigoplus_{i=0}^{n} A_i x(k+1-i) \;\oplus\; \bigoplus_{j=0}^{m} B_j u(k+1-j)\,,$$
$$y(k) = C_0 x(k)\,.$$

Provide matrices A_i, B_j, and C_0, for all relevant i and j.

b) (2 points)
Obtain matrix A_0^*, justifying how you perform the calculations.

c) (5 points)
We now want to represent the above equations in the state-space form

$$\widetilde{x}(k+1) = A\widetilde{x}(k) \oplus Bu(k+1)\,,$$
$$y(k) = C\widetilde{x}(k)\,.$$

How would you define $\widetilde{x}(k)$? Provide also the resulting matrices A, B, and C in terms of matrices A_i, B_j, and C_0. (You do not have to explicitly compute each element of matrices A, B, and C.)