# Questionaire on

Subject of Examination (legible if possible ;) )
Foundations of Stochastic Processes - Professor Caire.

☒ Oral ○ Written ○ Oral Reeximination

Date: 23.02.2022 Duration: 30 minutes

Examiner: Professor Caire Programme of Study: Elektrotechnik

**Preparation**

a) Continuous attendance at lectures? ☒ Yes ○ No

b) Effects of a): ☒ Positive ○ None ○ Negative

c) Amount of time spent on preparation: ~80h ☒ by yourself ☒ group work

d) Prior knowledge from other lectures/practical experiences?
Bachelor courses on stochastics and communication

e) What resources did you use? (*literature, websites etc.*)
I mainly used the materials provided by the professor - lecture slides, problem sheets and the example questions for the oral exam

f) Can you give any advice on the preparation of this exam?
The questions in the exam are, from what I can judge, very similar to the example questions the professor provided during the last week of lectures. It is important to understand the concepts, but the solutions provided in the problem sheets are more complex and advanced than what is going to be asked in the exam.

**Exam**

a) Had there been any agreements on form or contents of the exam? Were they met?
The professor gave out 2 questions, 10 mins for preparing them and 20 min presenting them on the board

b) Advice on behaviour during the exam:
The 10 minutes preparation are not enough to fully answer all questions, it is more important to understand the questions and think about how to solve them.

c) Examination style: (*atmosphere, questions: clear or unclear, in depth knowledge or general questions, specific interposed questions, specific questions in case of knowledge gaps, ...?*)
The professor was more helpful than I would have expected. The questions seem to be very similar to the example exam questions he provided, so being able to solve them is a good preparation for the exam.
During the exam it seems important to show understanding of the concepts. When you struggle with some little things the professor gives hints, which do not seem to influence the grade very much.

**Other questions**

a) How were you graded? (*optional of course*) 1.3

b) Do you think this grade is appropriate? ○ Yes ○ No (*why not?*)

c) Would you recommend this exam? ○ Yes (*to whom especially?*) ☒ No (*why not?*)
Lots and lots of difficult topics are covered in a very short time, problem sheets are way too complex

d) Do you have any other advice or remarks about this exam?

**Contents of the Exam:** Please try to reproduce as many questions as possible. At which points did the examiner ask for derivations, at which for analytic proof? (*If the space here is not sufficient do not hesitate to add additional sheets. But please staple the pages and number them.*)

1. Problem (Linear Estimation)
Let $\{X_n\}$ denote a WSS process with mean zero and power spectral density given by

$P_x(f) = \exp(-\cos(\pi f))$ , $f \in [-1/2, 1/2]$

a) Find the expression of the MMSE one-step prediction error
b) Suppose now that we observe $Y_n = X_n + Z_n$ where $Z_n$ is an i.i.d. zero-mean process with $E[(Z_n)^2] = \delta$ . Find the expression of the MMSE causal estimation error for estimating $X_{n+1}$ from $\{Y_m : m \le n\}$ .

2. Problem (Markov Chains)
We have a bag with m different numbers. At each time instant n we pick a number $N_n$ uniformly randomly from the bag and then return it to the bag again. We denote the number of different numbers we have observed until n by $X_n$

a) Show that $X_n$ is a Markov Chain and specify the transition probabilities. (Hint: you can use the state space $S=1,2,...,m$ where state $i \in S$ corresponds to observing i different numbers.)
b) Is the chain irreducible? If not specify the class of transient states
c) Let $T_i$, $i \in S$, be the average time we need to wait before observing all m numbers provided that we have already observed i of them. Show that $T_i = T_{i+1} + m/(m-i)$ (Hint: apply the Markov Property and the fact that $T_m = 0$)
d) Show that the average time before observing all the numbers is approximately $m \log(m)$. (Hint: you may find the formula $\sum_{i=1}^{m} 1/i \sim \log(m)$ useful)

**Thank you for your help!** **Your fellow students.**