V1)

$$\mathcal{L}[t^2 \sin t](s) = \left(-\frac{d}{ds}\right)^2 \frac{1}{s^2+1} \quad 2$$

$$= \left(\frac{-2s}{(s^2+1)^2}\right)' = \frac{-2}{(s^2+1)^2} + \frac{8s^2}{(s^2+1)^3} \quad 1 \quad = \frac{6s^4+4s^2-2}{(s^2+1)^4}$$

$$\mathcal{L}[\cos 4t \, e^{-2t}](s) = \mathcal{L}[\cos 4t](s+2) \quad 1$$

$$= \frac{s+2}{(s+2)^2+16} \quad 2$$

$$\mathcal{L}\left[\int_0^t \sin(3(t-u)) \cos u \, du\right] = \mathcal{L}[\sin 3t * \cos t](s) \quad 1$$

$$= \mathcal{L}[\sin 3t](s) \cdot \mathcal{L}[\cos t](s) \quad 1$$

$$= \frac{3}{s^2+9} \cdot \frac{s}{s^2+1} \quad 2$$

V3) f

w

w

f

f

V2)

$$\hat{f}(\omega) = \int_{-\infty}^{+\infty} f(t)\, e^{-i\omega t}\, dt$$

$$= \int_{-\infty}^{0} f(t)\, e^{-i\omega t}\, dt + \int_{0}^{\infty} f(t)\, e^{-i\omega t} \quad 2$$

$$= \int_{0}^{\infty} f(-s)\, e^{i\omega s}\, ds \; 2 + \int_{0}^{\infty} f(t)\, e^{-i\omega t}\, dt$$

$$= \int_{0}^{\infty} f(s)\, e^{i\omega s}\, ds \; 1 + \qquad ''$$

$$= F(-i\omega) + F(i\omega) \quad 3$$

V3)

a) falsch f

b) wahr w

c) wahr w

d) falsch f

e) falsch f

$$Z'' - \lambda_2 Z = 0$$

$$\Phi'' - \lambda_3 \Phi = 0$$

$$r^2 R'' + r R' + (\lambda_3 - (\lambda_1 - \lambda_2) r^2) R = 0$$

4c) Ansatz $R(r) = y(\omega r)$

$\Rightarrow \quad R'(r) = \omega\, y'(\omega r)\,, \quad R''(r) = \omega^2\, y''(\omega r)\,,$

einsetzen in (2):

$$r^2\omega^2\, y''(\omega r) + r\omega\, y'(\omega r) + ((\omega r)^2 - \ell^2)\, y(\omega r) = 0\,,$$

das ist die Besselsche DGL mit $x = \omega \cdot r$

$\Rightarrow \quad y(x) = J_\ell(x) = J_\ell(\omega r) = R(r)$ ist Lösung 2

$R(a) = J_\ell(\omega a) \overset{!}{=} 0 \quad \Rightarrow \quad \omega = \omega_{\ell k}\,,$ die

k-ten Nullstellen der ℓ-ten Besselfunktion.

Also $R(r) = J_\ell(\omega_{\ell k} \cdot r)\,.$ 2

R 1) $Y(s) = \mathcal{L}[y](s)$

$$s^2 Y(s) - s\underbrace{y(0)}_{1} - \underbrace{y'(0)}_{0} - 4Y(s) = \frac{1}{2}\left(\frac{1}{s-1} - \frac{1}{s+1}\right)$$

$$= \frac{1}{s^2-1} \qquad 3$$

$$\Rightarrow (s^2-4)\,Y(s) = \frac{1}{s^2-1} + s$$

$$\Rightarrow Y(s) = \frac{1}{(s^2-1)(s^2-4)} + \frac{s}{s^2-4} \qquad 2$$

$$= -\frac{1}{3}\left\{\frac{1}{s^2-1} - \frac{1}{s^2-4}\right\} + \frac{s}{s^2-4} \qquad \cancel{3}\,2$$

$$\Rightarrow y(t) = -\frac{1}{3}\left\{\sinh t - \frac{1}{2}\sinh 2t\right\} + \cosh 2t \qquad 3$$

$$\frac{1}{12}\left(e^{+2t} - e^{-2t}\right) + \frac{1}{2}\left(e^{2t} + e^{-2t}\right) - \frac{1}{6}\left(e^{t} - e^{-t}\right)$$

$$\frac{7}{12}e^{2t} + \frac{5}{12}e^{-2t} - \frac{1}{6}e^{t} + \frac{1}{6}e^{-t}$$

T2 a)

$$\left(\frac{du}{dt}\right)^\wedge (k,t) = \frac{d\hat{u}}{dt}(k,t)$$

$$\left(\frac{d^2u}{dx^2}\right)^\wedge (k,t) = -k^2\,\hat{u}(k,t) \qquad 3$$

$$\Rightarrow \quad \frac{d\hat{u}}{dt} = -k^2\,\hat{u} \qquad 3$$

$$\Rightarrow \quad \hat{u}(k,t) = c \cdot e^{-k^2 t} \qquad 2$$

mit $c = \hat{u}(k,0) = \left(e^{-\frac{x^2}{4}}\right)^\wedge (k) \quad a = \frac{1}{2}$

$$= \sqrt{4\pi}\, e^{-k^2} \qquad 2$$

$$\Rightarrow \quad \hat{u}(k,t) = \sqrt{4\pi}\, e^{-(t+1)k^2} \quad a = \frac{t+1}{2}$$

$$\Rightarrow \quad u(x,t) = \sqrt{4\pi} \cdot \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{\sqrt{2(t+1)}}\, e^{-\frac{x^2}{4(t+1)}}$$

$$= \frac{1}{\sqrt{t+1}}\, e^{-\frac{x^2}{4(t+1)}} \qquad 3$$

(24)

$$\frac{x}{2}\left(1+\frac{1}{t^2}\right) e^{\frac{x}{2}\left(t-\frac{1}{t}\right)} = \sum_{k=-\infty}^{+\infty} J_k(x) \cdot k\, t^{k-1} \qquad 3$$

Mit $t=1$: 1

$$x = \sum_{k=-\infty}^{+\infty} J_k(x) \cdot k \qquad 2$$

$$= \sum_{k=-\infty}^{-1} k\, J_k(x) + 0 + \sum_{k=1}^{\infty} k \cdot J_k(x)$$

$$\overset{l=-k}{=} \sum_{l=1}^{\infty} (-l)\, J_{-l}(x) + \sum_{k=1}^{\infty} k \cdot J_k(x)$$

$$= \sum_{l=1}^{\infty} (-l)(-1)^{l} J_l(x) + \text{''}$$

$$= \sum_{k=1}^{\infty} k\left[(-1)^{k+1} + 1\right] J_k(x) \qquad 3$$

$$= 2 \sum_{\substack{k=1 \\ k \text{ ungerade}}}^{\infty} k\, J_k(x)$$

$$= 2 \sum_{n=0}^{\infty} (2n+1)\, J_{2n+1}(x) \qquad 1$$