First Memory Log – ReSyst

WiSe 2022/2023

Tipp:

MTV reuses prompts between exams that take place on the same day.

1st Prompt

Determine the LTS for the following CCS processes:

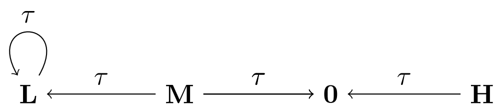
$$\mathbf{H} \overset{\text{def}}{=} \tau.\mathbf{0}$$
$$\mathbf{L} \overset{\text{def}}{=} \tau.\mathbf{L}$$
$$\mathbf{M} \overset{\text{def}}{=} \mathbf{H} + \mathbf{L}.$$

Solution

Let $\mathsf{Act} = \{\tau\}$. The LTS to the given CSS processes is T_1 with

$$T_1 = (\mathsf{Proc}_1, \mathsf{Act}, \mathsf{Tran}_1) = (\{\mathbf{H}, \mathbf{L}, \mathbf{M}, \mathbf{0}\}, \mathsf{Act}, \{(\mathbf{H}, \tau, \mathbf{0}), (\mathbf{L}, \tau, \mathbf{L}), (\mathbf{M}, \tau, \mathbf{0}), (\mathbf{M}, \tau, \mathbf{L})\}).$$

Or graphically:



2nd Prompt

We define

$$\mathbf{A}_0 \overset{\text{def}}{=} \mathbf{0}, \text{ and}$$
$$\mathbf{A}_{i+1} \overset{\text{def}}{=} \tau.\mathbf{A}_i, \text{for all } i \in \mathbb{N}^+.$$

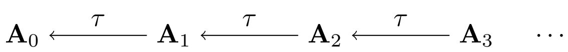
Determine the LTS for these CSS processes.

Solution

The LTS to the given CSS process is given by $T_2 = (\mathsf{Proc}_2, \mathsf{Act}, \mathsf{Tran}_2)$ with:

$$\mathsf{Proc} = \{\mathbf{A}_i \mid i \in \mathbb{N}\}$$
$$\mathsf{Tran} = \{(\mathbf{A}_i, \tau, \mathbf{A}_{i-1}) \mid i \in \mathbb{N} \wedge i \geq 1\}.$$

Or graphically:



3rd Prompt

Now extend the LTS from the second prompt to include **A** where

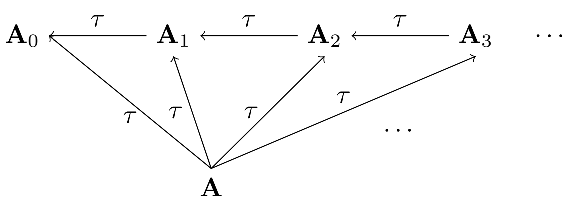
$$\mathbf{A} \overset{\text{def}}{=} \sum_{i \in \mathbb{N}} \tau.A_i.$$

Solution

The LTS to the given CSS process is $T_3 = (\mathsf{Proc}_3, \mathsf{Act}, \mathsf{Tran}_3)$ with:

$$\begin{aligned} \mathsf{Proc}_3 &= \mathsf{Proc}_2 \cup \{\mathbf{A}\} \\ \mathsf{Tran}_3 &= \mathsf{Tran}_2 \cup \{(\mathbf{A}, \tau, \mathbf{A}_i) \mid i \in \mathbb{N}\}. \end{aligned}$$

Or graphically:



4th Prompt

Now extend the LTS from the third prompt to include **I** where

$$\mathbf{I} \overset{\text{def}}{=} \mathbf{A} + \mathbf{L}.$$

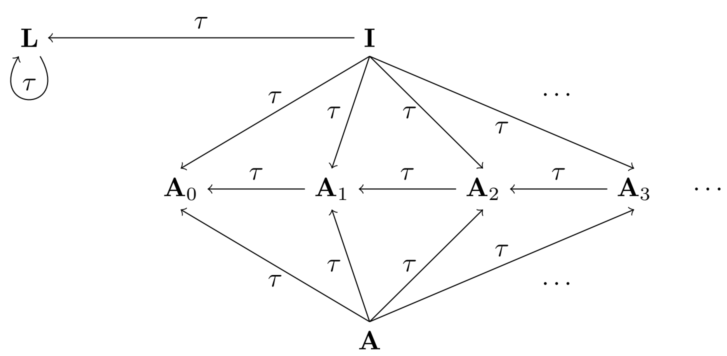
And **L** has the same definition as in the first prompt.

Solultion

The LTS to the given CSS process is $T_4 = (\mathsf{Proc}_4, \mathsf{Act}, \mathsf{Tran}_4)$ with:

$$\begin{aligned} \mathsf{Proc}_4 &= \mathsf{Proc}_3 \cup \{\mathbf{L}, \mathbf{I}\} \\ \mathsf{Tran}_4 &= \mathsf{Tran}_3 \cup \{(\mathbf{I}, \tau, \mathbf{A}_i) \mid i \in \mathbb{N}\} \cup \{(\mathbf{B}, \tau, \mathbf{L}), (\mathbf{L}, \tau, \mathbf{L})\}. \end{aligned}$$

Or graphically:



5th Prompt

Does $\mathbf{A} \sim \mathbf{I}$ hold in the LTS described in the fourth prompt.

Solution

It does not hold.

6th Prompt

Prove that $\mathbf{A} \sim \mathbf{I}$ dose not hold using gametheory.

Solution

We prove this statement by describing a winning strategy for the attacker. The attacker starts with the move $(\mathbf{A}, \mathbf{I}) \rightarrowtail ((\mathbf{A}, \mathbf{L}), (\tau, r))$. The defender then has to respond with $((\mathbf{A}, \mathbf{L}), (\tau, r)) \rightarrowtail (\mathbf{A}_i, \mathbf{L})$, for some $i \in \mathbb{N}$. The attacker can force the defender into a winning position in i rounds by repeatably using $\mathbf{L} \xrightarrow{\tau} \mathbf{L}$ to force the defender into defending $((\mathbf{A}_0, \mathbf{L}), (\tau, r))$.

7th Prompt

Does $\mathbf{A} \approx \mathbf{I}$ likewise not hold?

Solution

No, $\mathbf{A} \approx \mathbf{I}$ holds. The attacker can only make moves using τ since $\mathsf{Act} = \{\tau\}$, the defender can always respond to such a move, since $p \overset{\tau}{\Rightarrow} p$ holds for all $p \in \mathsf{Proc}_4$.

8th Prompt

Find a distinguishing HML formula for $\mathbf{I}$ and $\mathbf{A}$.

Solution

$\mathbf{I}$ and $\mathbf{A}$ are distinguished by

$$\begin{aligned} &F = X \\ &X \overset{\max}{=} \langle \tau \rangle X. \end{aligned}$$

9th Prompt

Determine the semantics of F.

Solution

$$[\![F]\!] = \mathrm{FIX}\, \mathcal{O}_{\langle \tau \rangle X}$$

We therefore need to find the greatest fixpoint of the complete lattice $(2^{\mathsf{Proc}_4}, \subseteq)$ with regards to the monotonic function $\mathcal{O}_{\langle \tau \rangle X}$. Since this lattice is not finite we cannot use the fixpoint theorem of Knaster and Tarski to guaranteed that we can find a fixpoint.

None the less we can observe that the repeated application of $\mathcal{O}_{\langle \tau \rangle X}$ on Proc_4 removes a $\mathbf{A}_i$ with each step. From this we conclude, what we can also intuitively see, that $[\![F]\!] = \{\mathbf{L}, \mathbf{I}\}$.

10th Prompt

Why is this a fixpoint?

Solution

Because $\mathcal{O}_{\langle \tau \rangle X}(\{\mathbf{L}, \mathbf{I}\}) = \{\mathbf{L}, \mathbf{I}\}$.

11th Prompt

Why is this the greatest fixpoint?

Solution

Intuitively we can see that $\mathbf{A}_0 \notin P$ if P is a fixpoint of $\mathcal{O}_{\langle \tau \rangle X}$. And that $\mathbf{A}_i \in P \implies \mathbf{A}_{i-1} \in P$ for all $i \in \mathbb{N}$ and $i > 1$ if P is a fixpoint.

$\{\mathbf{L}, \mathbf{I}\}$ is therefore the greatest fixpoint.