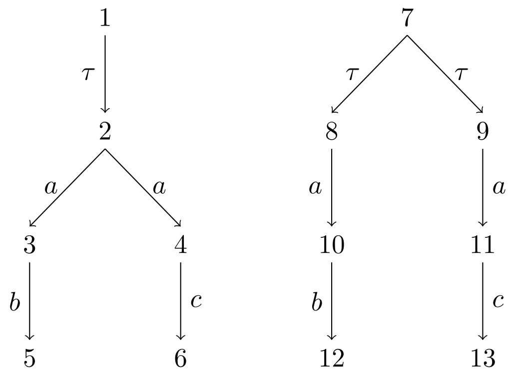
Second Memory Log – ReSyst

WiSe 2022/2023

Tipp:

MTV reuses prompts between exams that take place on the same day.

1st Prompt



How does the process 1 relate to 7, e.g. $\sim$, $\approx$, $\lesssim$, and $\lessapprox$?

Solution

$1 \nsim 7$, $1 \not\approx 7$, $1 \not\lesssim 7$, $1 \not\lessapprox 7$ but $7 \lesssim 1$ $7 \lessapprox 1$.

2nd Prompt

Tell us something about the relation "$\sim$".

Solution

The bisimilarity relation $\sim$ describes which process pairs are part of a bisimulation. The bisimilarity relation is furthermore the union of all bisumlations and thus the largest bisumlation.

3rd Prompt

Determine $\sim$ for the LTS given in the first prompt.

Solution

Working from the bottom up we can see that

$$\sim = \{5,6,12,13\}^2 \cup \mathrm{r}(\mathrm{s}(\{(3,10),(4,11)\})).$$

3rd Prompt

How else could you have determined $\sim$?.

Solution

Using the fact that the LTS is finite and the fixpoint theorem of Knaster and Tarski we can compute the bisimilary as the greatest fixpoint of the monotonic function $\mathcal{F}$ in the complete lattice $(2^{\mathsf{Proc}\times\mathsf{Proc}}, \subseteq)$.

1 4th Prompt

How would you show that $\mathcal{F}$ is monotonic?

Solution

$\mathcal{F}$ is monotonic iff. $A \subseteq B \implies \mathcal{F}(A) \subseteq \mathcal{F}(B)$. We now assume $A \subseteq B$ and $(p,q) \in \mathcal{F}(A)$. Since $(p,q) \in \mathcal{F}(A)$ we know that

$$\forall \alpha \in \mathsf{Act}.\forall p' \in \mathrm{Der}(p,\alpha).\exists q' \in \mathrm{Der}(q,\alpha).(p',q') \in A$$
$$\forall \alpha \in \mathsf{Act}.\forall q' \in \mathrm{Der}(q,\alpha).\exists p' \in \mathrm{Der}(p,\alpha).(p',q') \in A.$$

Since $A \subseteq B$ we can conclude that

$$\forall \alpha \in \mathsf{Act}.\forall p' \in \mathrm{Der}(p,\alpha).\exists q' \in \mathrm{Der}(q,\alpha).(p',q') \in B$$
$$\forall \alpha \in \mathsf{Act}.\forall q' \in \mathrm{Der}(q,\alpha).\exists p' \in \mathrm{Der}(p,\alpha).(p',q') \in B.$$

Therefor $(p,q) \in \mathcal{F}(B)$.

5th Prompt

What are the semantics of least fixpoint of $\mathcal{F}$ in this lattice?

Solution

The least fixpoint contains all pairs of processes $p \in \mathsf{Proc}$ where for all $\alpha \in \mathsf{Act}$ $\mathrm{Der}(p,\alpha) = \{\}$. It also contains all parents of these processes that are bisimilar.

6th Prompt

Determine this least fixpoint.

Solution

Using the approach described in the solution to the third prompt, but for the least fixpoint, we can observe that for this specific case the least and greatest fixpoint are one in the same.

7th Prompt

Find a HML formula that distinguishes 1 and 7.

Solution

The HML formula $F = \langle \tau \rangle [\, a \,] [\, b \,] \mathit{ff}$ distinguishes 1 and 7.

8th Prompt

Determine F^c.

Solution

$$F^c = [\, \tau \,] \langle a \rangle \langle b \rangle \mathit{tt}$$

9th Prompt

Dose this formula also distinguish 1 and 7? If so why?

Solution

Yes it does. Since the semantics of $\cdot^c$ where chosen such that $[\![\, P^c \,]\!] = \mathsf{Proc} \setminus [\![\, P \,]\!]$.

10th Prompt

Is there a shorter formula that distinguishes 1 and 7? Why?

Solution

No there is no such formula. This can be seen by observing that $1 \sim_2 7$. Using the Hennesy-Milner-Theorem we can conclude that $[\![\, 1 \,]\!]^{\leq 2} = [\![\, 7 \,]\!]^{\leq 2}$. A distinguishing formula P therefore has $\mathrm{mt}(P) \geq 3$.

11th Prompt

Find a CCS formula that describes the behaviour of the process 1, that does not use a τ-prefix.

Solution

$$\big((d.a.(b.\mathbf{0} + c.\mathbf{0})) \mid \overline{\mathrm{d}}.\mathbf{0}\big) \setminus \{\, \mathrm{d} \,\}$$