GSNT – SS2012

Covarianz $C_{xy} = E[(X-\mu_x)(Y-\mu_y)]$

weißes Rauschen $\sigma_x^2 \cdot \delta(k)$

Bernoulli-Formel

$$B(n,p,k)\binom{n}{k}p^k(1-p)^{n-k}$$

n – Anzahl Exp.
k – Anzahl Treffer
p – Wahrsch. f. Treffer

W-Hopf-Gl. für

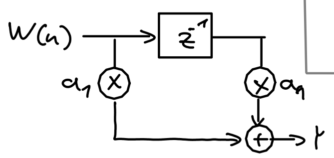
$$R_{xx} \cdot h_{opt} = r_{xx}$$

AKF-Matrix, opt. FK

$$P(B|A) = \frac{P(A\cap B)}{P(A)}$$

Moving-Average-Filter

$$Y(n) = a_0 W(n) + a_1 W(n-1)$$
$$= W(n)\cdot(a_0 + a_1 z^{-1})$$



$$R_{YY}(k) = E[Y(n)\cdot Y(n+k)]$$
$$\hookrightarrow \; = E[(a_0 W(n) + a_1 W(n-1))(a_0 W(n+k) + a_1 W(n+k-1))]$$

→ ausmultiplizieren → $a_0^2 R_{WW}(k) + a_0 a_1 R_{WW}(k+1) R_{WW}(k-1) + a_1^2 R_{WW}(k)$
→ mit weißem Rauschen $R_{WW}(k)=0$, $R_{WW}(0)=\sigma_x^2$ → Tada!

Bayes'sche Regel

$$P(A|B) = \frac{P(A)}{P(B)}\cdot P(B|A)$$

$$\binom{n}{k} = \frac{n!}{n!\cdot(n-k)!}$$

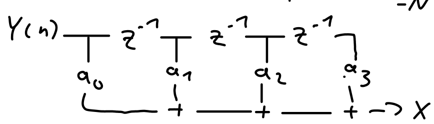
Wahrsch. aus bed. Wahrsch.

$$P(B) = \sum_{j=1}^{N} P(A_j)\cdot P(B|A_j)$$

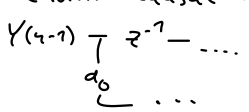
Arten von W-K-Filtern

1) unendlich kausal: $\int_{-\infty}^{0} y \ldots$ geht nicht

2) endlich kausal, N=3: $\int_{-N}^{0}$



3) endlich kausal mit Verz. $\int_{-N}^{M}$



4) nicht kausal: $\int_{-\infty}^{\infty}$ n.i.

Zufallsvariablen als Vektoren

$\|X\| = E[X^2]$ Skalarprodukt

$(X,Y) = E[X\cdot Y] = \|X\|\cdot\|Y\|\cdot\cos(\angle(X,Y))$

Matrix invertieren

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \xrightarrow{-1} \frac{1}{ad-cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

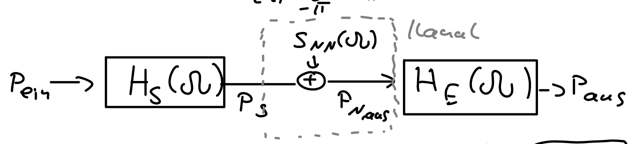
Ausgangsleistung

$$P_{aus} = \frac{1}{2\pi}\int_{-\pi}^{\pi} S_{xx}(\Omega)\,d\Omega$$

Störleistung

$$P_{N,aus} = \frac{1}{2\pi}\int_{-\pi}^{\pi} S_{NN}(\Omega)\cdot|H_E(j\Omega)|^2$$

$$SNR = \frac{P_{ein}}{P_{N,aus}} = \frac{\frac{1}{2\pi}\int_{-\pi}^{\pi} S_{xx}(\Omega)\,d\Omega}{\frac{1}{2\pi}\int_{-\pi}^{\pi} S_{NN}(\Omega)\cdot|H_E(j\Omega)|^2\,d\Omega}$$



$$|H_S(j\Omega)|^2 = \sqrt{\frac{S_{NN}(\Omega)}{S_{xx}(\Omega)}}$$

$$|H_E(j\Omega)|^2 = \sqrt{\frac{S_{xx}(\Omega)}{S_{NN}(\Omega)}}$$

Maß spektraler Konstanz

$$\gamma_x^2 = \frac{e^{\frac{1}{2\pi}\int_{-\pi}^{\pi} \ln(S_{XY}(\Omega))\,d\Omega}}{\sigma_x^2}$$

Weißes Rauschen

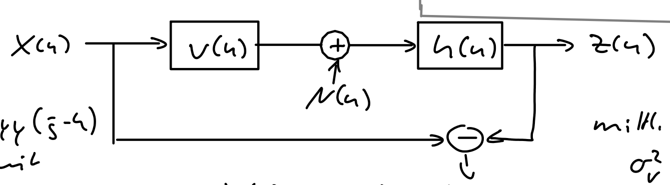
$R_{WW}(k) = \sigma_W^2\,\delta(k)$ MSK = 1

$S_{WW}(\Omega) = \sigma_W^2$

Vert.-Dichte-Fkt.

$$p_X(x) = \frac{\partial F_X(x)}{\partial x}$$

Wiener-Kolmogorow-Filter



$$X(n) - Z(n) = R(n)$$

$E[(X+Y)^2] = P_X + P_Y + 2E[X\cdot Y]$

mittl. quadr. Fehler (P_R)

$\sigma_V^2 = E[\{X(n) - Z(n)\}^2]$

$\hookrightarrow$ σ_V minimieren (lineare Schätzung)

Wiener-Hopf-GS

$$R_{XY}(j) = \sum_{k\in I} h_{k,opt}\, R_{YY}(j-k)$$

mit $j \in I$

Wiener-Hopf-Bedingung

$$R_{YX}(k) = R_{Y Z_{opt}}(k), \; k \in I$$

Orthogonalitätsprinzip

$$E[\{X(n) - Z_{opt}(n)\}\, Z_{opt}(n)] = 0$$

2.) $z(n) = \sum_{k=0}^{N} h_k\, y(n-k)$

5.) kausales Filter: $z(n) = \sum_{k=0}^{\infty} h_k\, y(n-k)$

arith. MW

$$\mu_X = E[X] = \int_{-\infty}^{\infty} x\, p_X(x)\,dx$$

$$E[X] = \sum_{k=1}^{N} P_k \cdot x_k$$

Leistung

$$P_X = E[X^2] = \int_{-\infty}^{\infty} x^2 p_X(x)\,dx$$

$$P_X = E[X^2] = \sum_{k=1}^{N} P_k x_k^2$$

Varianz

$$\sigma_X^2 = E[(X-\mu_X)^2] = \int_{-\infty}^{\infty} p_X(x-\mu_X)\,dx$$

$$E[(X-\mu_X)^2] = \sum_{k=1}^{N} P_k (x_k - \mu_k)^2$$

Bsp.: Filterkoeffizienten

X mit $E[X]=\mu_X$, Y mit $E[Y]=\mu_Y$

$\hat{X} = aY + b$ – lineare Schätzung

$Z = \hat{X} - X = aY + b - X$ – Leistung des Schätzfehlers

$$P_Z = R_{ZZ}(0) = \sigma_Z^2 - \mu_Z^2$$
$$= E[Z^2] = E[(aY+b)^2] - 2E[(aY+b)X] + E[X^2]$$
$$= a^2 E[Y^2] + 2ab\,E[Y] + b^2 - 2a\,E[XY] - 2b\,E[X] + E[X^2]$$

($E[Y^2] \to \sigma_Y^2$, $E[Y] \to 0$, $E[XY] \to 0$, $E[X] \to \mu_X$, $E[X^2] \to \mu_X^2$)

$$= a^2\sigma_Y^2 + b^2 - 2b\mu_X + \mu_X^2 = P_Z$$

minimieren durch Abl.

$$\frac{\partial E[Z^2]}{\partial a} = 2a\sigma_Y^2$$

$\Rightarrow a_{opt} = 0$

$$\frac{\partial E[Z^2]}{\partial b} = 2b - 2\mu_X$$

$\Rightarrow b_{opt} = \mu_X$

diskrete Zufallsvariable

$$F_X(x) = \sum_{k=1}^{N} P_k\, \sigma(x - x_k)$$

$$p_X(x) = \sum_{k=1}^{N} P_k\, \delta(x - x_k)$$

Transformation von ZV

$Y = g(X)$, g monoton

$$p_Y(y) = \left.\frac{p_X(x)}{|g'(x)|}\right|_{x=g^{-1}(y)}$$

AKF

$$R_{XX}(k) = E[X(n)\cdot X(n+k)]$$

KKF

$$R_{YX}(k) = E[X(n)\, Y(n+k)]$$

LDS

$$S_{XX} = \sum_{k=-\infty}^{\infty} R_{XX}(k)\, e^{-jk\Omega}$$

$y(n) = x(n) * h(n)$

$$= \sum_{-\infty}^{\infty} h(k)\cdot x(n-k)$$

Autokovarianz

$$C_{XX}(k) = E[(X(n)-\mu_X)\cdot(X(n+k)-\mu_X)]$$

ergodischer Prozess

- Gesamt-MW = Zeit-MW
- Stationarität (MW gilt für jedes n bei X(n))
- stat. Kenngröße durch eine Musterfolge